



# Pioneers of the Ice Age Models: A Brief History from Agassiz to Milankovitch

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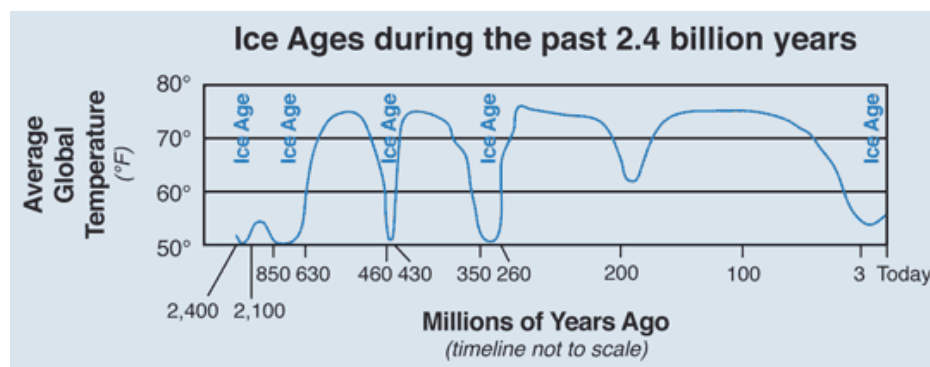
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**Abstract.** It is currently known that astronomical factors trigger the emergence of glacial and interglacial periods. However, nearly two centuries ago, the overall situation was not as apparent as it was with today's scientists. In this article, I briefly discuss the astronomical model of ice ages put forward between the 19th and 20th centuries. This century was indeed *annus mirabilis* for scientists to understand the ice age phenomenon. Agassiz, Adhémar and Croll laid the foundation stones for understanding the dynamics of ice ages. But it was Milankovitch who combined empirical geology with mathematical astronomy. To put specifically, he identified the shortcomings of the preceding ice age models and modified his model accordingly. In what follows, I review former approaches to the ice age problem and show how they failed to meet their objectives. Next, I show how Milankovitch's model managed to capture all sufficient astronomical elements. Last sections focus on Milutin Milankovitch's genuine approach, including his accomplishment of tackling the problem mathematically.

## 15 1 The Problem

There seems no strict quantitative definition of ice ages (see Davis, 2001; Barry and Yew Gan, 2011, p.299). However, we may easily say that ice ages are periods when severe temperature reduction occurs across the Earth. During such periods of time, surface of the Earth is largely covered with ice sheets and mountain glaciers. So far, scientists provide evidence that the Earth, in its history, has experienced many glacial periods. However, only few of these have established dominance over the whole climate system by lasting more than millions of years. The chart below shows these extensive ice ages in the past 2.4 billion years (Fig. 1).





25 **Figure 1: The chart shows five major ice ages during the past 2.4 billion years. Reprinted from Utah Geological Survey Notes, Eldredge S. and Biek B. (2010) [Adapted from Saltzman (2002)], Retrieved from [https://geology.utah.gov/wp-content/uploads/ice\\_ages1](https://geology.utah.gov/wp-content/uploads/ice_ages1). © Copyright – Utah Geological Survey – State of Utah. May, 15, 2017.**

As seen from the chart, scientists have identified five significant ice age periods during the past 2.4 billion years. They are respectively called Huronian (2.4–2.1 billion years ago), Cryogenian (850–630 million years ago), Andean–Saharan (460–430 million years ago), Karoo (360–260 million years ago) and Quaternary (2.6 million years ago–present). Accordingly, we  
30 are living in the Quaternary period which began approximately 2.6 million years ago. Since we live in the Quaternary period, we are currently in the last major ice age period. So, the Earth should be in a notably colder climate compared to our current condition. At present, however, this is not the case. Neither climate is extremely cold, nor are our continents largely covered by ice sheets. On what grounds, then, scientists claim that we are in a major ice age period?

Geologists already have the answer to this question. It seems now the received view that the Earth currently is in an  
35 interglacial period that approximately have started about 15 thousand years ago (Lisiecki and Raymo, 2005). According to this view, our planet, sometime during an extensive ice age period, makes alterations toward warmer climatic conditions. After witnessing warmer climate conditions, it again goes into a deep freeze and this specific event recurs at intervals. Thus, in short, our climate is subject to certain periodical changes. With respect to time, these alterations are small steps for the Earth, but giant steps for mankind.

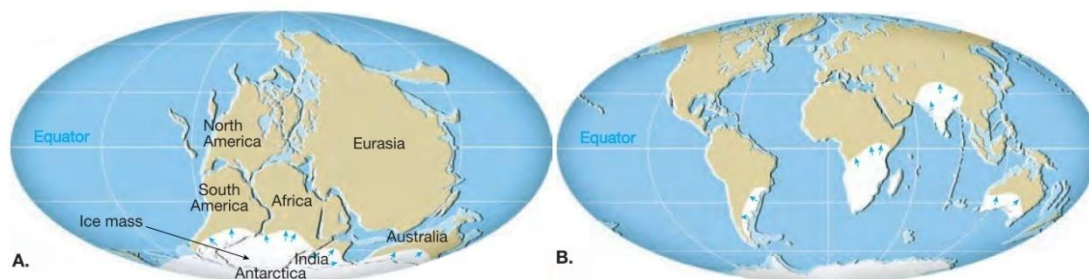
40 At first sight, the intuitive way to evaluate the alterations between glacial and interglacial periods is to assume a probable mechanism behind it. The scientists did so as well. Whether periodically or not, the Earth has witnessed, and probably will continue to witness numerous glacial and interglacial periods. Thus, for scientists, the new task is to find out which factors are responsible for this. The common questions which they had dealt with were as follows: Why the Earth's climatic conditions change within major ice ages? What factors are involved in the separation of glacial and interglacial periods?  
45 Why series of glacial and interglacial intervals are either longer or shorter than their previous ones? The remainder of this chapter will be concerned with these questions and the answers given to them, but before proceeding let me say few words about the occurrence of major ice ages.

## 2 Major Ice Ages

There are many causes of major glaciation, including oceanic fluctuations, volcanic eruptions or surface albedo (see also  
50 Rohling *et al.*, 2012). Among all these, however, the main cause responsible to initiate an ice age period is plate tectonics. Alfred Wegener (1880–1930), the German geologist and meteorologist was the first to come up with this hypothesis. He spent his time primarily in Greenland and his field research had mainly focused on continental drifts that led him to develop the revolutionary theory of plate tectonics. Although his purpose was not peculiarly aimed to find out the mechanism of glacial periods, his theory brought together a useful explanation for how these periods occur.



55 Given this theory, the separate continents today were once combined together. For instance, the well-known supercontinent Pangaea was one of them. It existed approximately between 350–250 million years ago and different from today's positioning, its continental mass was mostly located in southern hemisphere of the Earth. The east part of South America and the west part of Africa were bind together where India was beneath them. After a considerable time, each of these landmasses broke apart and drifted away until reaching their current positions (Fig. 2).



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**Figure 2: A. The supercontinent Pangaea existed approximately between 350–250 million years ago. Ice sheets are located close to the south pole of the Earth. B. Present position of the continents. Old ice sheet evidences are located on different land masses. Reprinted from Essentials of Geology (p. 282) by Lutgens, F.K., Tarbuck, E. J. and Tasa, D., 2012, U.S.A.: Prentice Hall. Copyright ©2012, 2009, 2006 by Pearson Education, Inc.**

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This theory has much to say about geological phenomena. With its principles, for example, we can deduce hypotheses about other natural events, such as volcanic eruptions, earthquakes or processes of mountain formation. Eventually, all these phenomena are results of the movements of plates which are segments of the continents and the oceans.

Similarly, the beginnings and the ends of major glacial periods are also related with the movements of continents. Many relevant textbooks give a clue about this relationship by using the term ‘continental glacier’ because glaciers (whether they are ice sheets or ice caps) can only form on ground. This information is crucial for the formation of glaciers, but it is also crucial for the explanation of the initial conditions of glacial periods. In order to understand this, it’s sufficient to consider that the positions of continents have decisive influence over the global climate. To put it precisely, it happens as follows: When the continents are positioned mostly near the poles, low amount of sunlight falls on a big portion of total continental area. This means that snow and ice accumulate over large areas. Hence, the albedo increases and significant part of sunlight returns back to space. As a result, global temperature decreases and ineluctably produces a glacial period. That’s why “many scientists suggest that ice ages have occurred only when Earth’s shifting crustal plates have carried the continents from tropical latitudes to more poleward positions” (Lutgens *et al.*, 2012, p.281).

This natural process can be exemplified by reviewing the structure of the supercontinent Pangaea. As I mentioned above, large portion of Pangaea continent was located close to the south pole of the Earth. This means that suitable conditions were provided for a glacial period to start. Today, based on paleoclimatological evidence, we know that the Earth has experienced an extensive ice age about 350–250 million years ago, also the era when Pangaea existed. This can also be seen in (Fig .1), where one of the lowest points of the curve denotes this period.



At this point, it is especially important to note that the theory of plate tectonics has no room or explanation for the questions raised at the end of the previous section. It is because this theory accounts for the long-term changes of global climate and falls short of establishing short-term temperature variations in major ice ages. According to the theory of plate tectonics, plates move so slowly (1–1.5 centimeters per year on average), and due to this fact, continents which are parts of plates move slowly too. This basic mechanism of the drift is extremely gradual such that continents shift their positions over millions of years. So, any model deduced from the theory of plate tectonics appears to be capable of explaining long term climatic changes. However, it would unavoidably fail to provide an explanation of what factors may be responsible for our climate to warm up and regress back to a cold climate within major ice age periods. “Therefore, we must look to some other triggering mechanism that may cause climate change on a scale of thousands rather than millions of years” (Lutgens *et al.*, 2012, p.282).

### 3 An Astronomical Solution for a Geological Puzzle

As described above, climatic changes within the ice age periods are not caused by movements or positioning of continents. For this reason, we need to seek the causes of these temperature changes, elsewhere, in the Earth system. Milutin Milankovitch (1879–1958), the Serbian mathematician, astronomer and engineer, thought so and came up with an idea to connect climatic changes with Earth’s orbital variations. According to him, temperature changes during the major ice age periods depend on the different amounts of solar irradiance falling on the surface of the Earth. By all means, the amount of solar irradiance is driven not by the Earth’s internal system but by an external force, namely the Sun. Therefore, a possible solution lies not only in the field of Geology, but also in the field of Astronomy.

The idea to understand the dynamics of ice ages from the astronomical perspective is not new, and hence it does not belong entirely to Milankovitch. There are other details which make Milankovitch an original figure in science. These details and his other contributions will be spelled out in more detail after. But first, I will outline briefly the pioneers of the astronomical/geological theory who influenced Milankovitch’s approach to the ice ages one way or another.

### 4 Pioneers of the Ice Age Theory

We know quite a lot about the glaciers and glaciation, such as how ice sheets move, how a glacier shrink and grow, or how glacial deposits are formed etc. The situation, however, was different over the past two centuries. In those days, few scientists were involved with the glaciation research. Moreover, many were not aware how crucial this natural phenomenon is in shaping the solid surface of the Earth.

The first sign of the ice ages was erratic boulders. These erratic boulders (large masses of rock) are found in places far from their bedrock source. Early to mid–18th century, the received view was that these giant rocks are somewhere distant from their original places because they are transported and deposited by a great flood –as almost told in the Bible. Although he



115 changed his mind later on, Charles Lyell (1797–1875), the British geologist, confidently defended such a view and accepted  
it as the most likely explanation for the emplacement and movement of boulders (Imbrie and Imbrie, 1979, pp. 21–22).  
While this view dominated the geological scene for over decades, it subsequently received criticisms from some researchers  
in the scientific community. The main objection urged against the flood theory was that such giant rocks are less likely to be  
carried from low to high elevations via floodwater. So, there must be another mechanism at work to transport massive  
boulders from one place to another. In addition, there was one other thing that made it hard for flood theorists to account for:  
120 scratched and grooved bedrock surfaces. These traces on the land surface were an indication of a serious problem because  
flood theorists were assuming a simple process of transportation of boulders from one place to another. However, in order to  
erode the underlying land surface such a way, boulders must strongly scrape against the ground that they move over.  
Therefore, floodwater alone was not a possible candidate to be the cause of scratched and grooved surfaces.

The time when the level of the erratic boulder discussion manifested itself in a puzzling way, Louis Agassiz (1807–1873),  
125 the Swiss biologist and geologist, was working on fossil fishes. However, along with fossil research, his mind was busy with  
another issue: the possible causal link between erratic boulders and glaciation. According to Agassiz, the boulders which we  
find far from their bedrocks are in their current different places not because they are carried by the flowing water but because  
they are removed by an agent with much stronger force. His hypothesis was that the displacement of a boulder from its  
bedrock could most probably be produced by the effect of glaciers.

130 In 1837, Agassiz gave a talk about his glacial theory to the Swiss Society of Natural Sciences at Neuchatel. The expectations  
have failed, because members of the society were prepared to hear a talk about his new research results on fossil fishes.  
Instead, Agassiz took the opportunity and announced his new argument on the phenomenon on glaciation. Truly, from that  
moment onwards the dispute had begun. Within the time, Agassiz was exposed not only to the criticisms of opponent camps,  
but also showed resistance to the career-oriented advices from the authorities of geology, including William Buckland  
135 (1784–1856) and Alexander von Humboldt (1769–1859). For example, von Humboldt once wrote in a letter:

I am afraid you work too much, and (shall I tell you frankly?) that you spread your intellect over too many  
subjects at once. I think that you should concentrate your moral and also your pecuniary strength upon this  
beautiful work on fossil fishes... In accepting considerable sums from England, you have, so to speak,  
contracted obligations to be met only by completing a work which will be at once a monument to your own  
140 glory and a landmark in the history of science... No more ice, not much of echinoderms, plenty of fish...  
(Quoted in Boylan, 1998, pp.146)

Despite such tough conditions, only about 30 years after, the theory suggesting the link between erratic boulders and  
glaciation was widely accepted in the scientific community, and this constituted a milestone in the advancement of glaciation  
research.

145 After being established as a physical process that occurred once or many times in the past, glaciation became a subject of  
geological investigations. Now the leading issue was no longer about the existence of past glacial periods. Instead, it was  
about the initial conditions of glaciations.



The French astronomer and mathematician Joseph Adh mar (1797–1862), was the first prominent scientist who drew attention to the onsets of glaciation process. The notable aspect of his contribution was his discovery of the relationship between global climate changes and Earth’s orbital variation. According to Adh mar, the precession of the Earth’s axis produces temporarily unequal seasons for each hemisphere. For this reason, one hemisphere enjoys longer summers, while the other experiences longer winters. The hemisphere which is in the latter case approaches to the process of glaciation. Therefore, he concluded that “whichever hemisphere had the longer winter would suffer an ice age” (Oldroyd, 1996, p.151). Another remarkable attempt to understand the onsets of glacial periods was made by the Scottish scientist and geologist James Croll (1821–1890). Croll highlighted the importance of seasonal insolation and its role in glaciation. According to him, the amount of sunlight that the Earth receives during the winters and summers is related to the Earth’s orbital parameters, namely the eccentricity and precession. These parameters play a crucial role in varying the amount of insolation. Due to these variations, the Earth’s climate either warms up or cools down. According to Croll “a decrease in the amount of sunlight received during the winter favors the accumulation of snow, and that any small initial increase in the size of the area covered by snow would be amplified by the snowfields themselves” (Berger, 2012, p.113). Therefore, the Earth would gradually suffer an ice age.

Agassiz, Adh mar and Croll are three significant figures that influenced the course of the glaciation research. However, they are not the only ones who deserve all the respect. There are many other scientists who contributed substantially to the field of ice age studies, by mainly observing nature over the years and making hypothesis about the past, present and future of the environmental conditions. Particularly, Imbrie and Imbrie (1979), Bard (2004), Berger (2012) and Paillard (2015), in their seminal works, traced the history of great glaciologists, their naturalist approaches and the breakthroughs in glaciation research, in detail. In the following section, however, I focus on Milankovitch whose contribution had a significant impact on the science of glaciation among all those researchers.

## 5 The Astronomical Model of Ice Ages

Like many scientists, Milankovitch stood on the shoulders of giants as well. The contribution he made to science, particularly to climatology was only possible with the past contributions of great scientists. In truth, Milankovitch often mentions the names of his predecessors in his papers, not only to proceed with their past research work, but also to give credit to their scientific legacy.

Nevertheless, as much as the past efforts should be recognized, we should also emphasize Milankovitch’s genuine approach to the problems. His approach is different and genuine from most of his predecessors because it involves some sort of unifying framework. Until the era of Milankovitch, the mainstream methodology on particular issues in geology was descriptive. However, for Milankovitch exact methodology (or in other words, mathematical approach) should be integrated into this descriptive stance. For him, only if this condition is met, we could hope to discover the geological traits of the Earth and other planets.





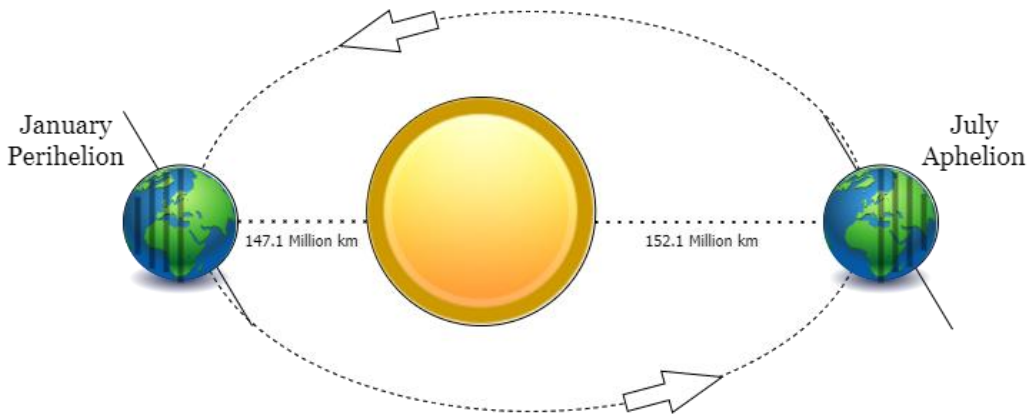
180 The time when Milankovitch encountered the problem concerning the glacial and interglacial periods, he was already  
equipped with this line of thought. According to him, those who were dealing with the issue had the necessary information to  
proceed, but they were not able to achieve it. Of course, there have been reasons for that. Given Milankovitch's  
interpretation, some of the scientists had little idea what to do with empirical evidence and how to relate them with relevant  
theory; while the others were unable to use their theoretical knowledge to construct the model of the investigated  
185 phenomenon (see Imbrie and Imbrie, 1979, pp. 97–99; Petrovic, 2012). In all this, Milankovitch was defending a unified  
view that suggests a combination of both approaches. Although, scientists like Croll made similar attempts to unite these  
approaches, they inevitably failed due to lack of empirical evidence or their insufficient mathematical training (see  
Milankovitch, 1941, p. 376).

As stated above, the way Milankovitch approached the problem was quite original. He argued that crucial climatic changes,  
190 such as series of glacial and interglacial intervals, are causally connected with the distribution of solar radiation that reaches  
the Earth's surface. Thus, any change in the amount of insolation triggers a change in the global climate. So, in order to  
understand much about the periods of glacial and interglacial climates, we must determine the factors that vary the amounts  
of insolation.

According to Milankovitch, the distribution of incoming solar radiation happens in accordance with the Earth's orbital  
195 variations. Though these orbital variations “cause little or no variation in the total solar energy reaching the ground”  
(Lutgens *et al.*, 2012, p. 282), they do play a role in seasonal changes. For example, they change the seasonal duration (long  
or short winters/summers) or the degree of difference between the seasons (hotter, milder or cooler winters/summers).  
Moreover, these orbital variations comprise three astronomical cycles: orbital eccentricity, axial tilt and precession. Most  
commonly they are called ‘Milankovitch Cycles’ in deference to the Serbian scientist.

## 200 **6 Milankovitch Cycles**

The first component of orbital variations is called orbital eccentricity. It can simply be described as the Earth's orbital path  
around the Sun. The gravitational tug of other large planets influences Earth's orbit. This leads to a change in orbital shape  
and hence produces a cycle lasting about 90000–100000 years. Actually, our planet appears to orbit around the Sun  
elliptically. However, the shape of this orbit is not fixed. Sometimes it becomes more elliptical and sometimes it becomes  
205 more circular. The orbit becomes more elliptical when the difference between the furthest and closest distance of the Earth  
from the Sun increases. Differently, when the difference between the furthest and closest distance of the Earth from the Sun  
reduces, the orbit becomes more circular. The place where the Earth is nearest to the Sun is called perihelion, which occurs  
around January 3. On or around July 4, however, the Earth is at its greatest distance from the Sun, which is called aphelion  
(Fig. 3).



210

**Figure 3: The distances from the Sun at perihelion and aphelion for the Earth.**

Orbital eccentricity takes a value obtained by a simple formula using the variables aphelion ( $a$ ) and perihelion ( $p$ ) as follows:

$$e = (a - p)/(a + p) \quad (1)$$

215 If the obtained value for orbital eccentricity is equal to zero ( $e=0$ ), the shape of the orbit is a perfect circle. If it is greater than zero ( $e>0$ ), then the shape of the orbit is an ellipse. When the current estimated values are inserted into the above equation, we get

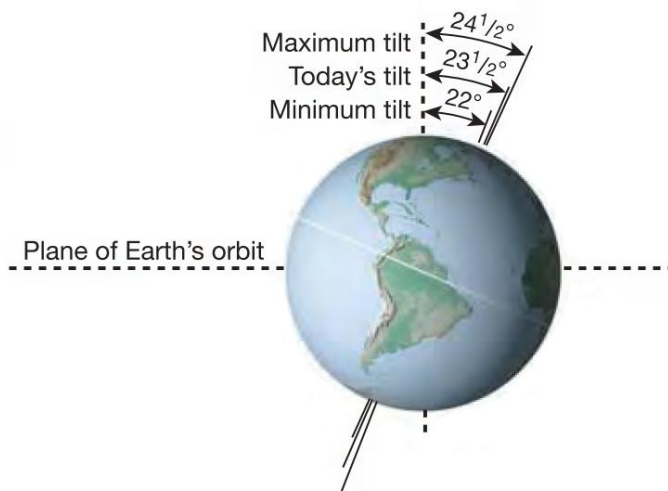
$$= (152.1 \times 10^6 - 147.1 \times 10^6)/(299.2 \times 10^6)$$

$$e = 0.0167112299$$

Orbital eccentricity has an influence on climate change. However, it is the least effective factor on glaciation, among other variations. The reason is that the eccentricity variations have small impact on total annual insolation, namely a difference of 220 0.03% (see Maslin and Ridgwell 2005, p. 21). Nevertheless, orbital eccentricity of the Earth may lead significant temperature contrast between seasons. For example, if the orbital path of the Earth around the Sun was circular, there would be no annual insolation difference between summer and winter for each hemisphere. But, due to gravitational effects of other planets, especially Jupiter's, this cannot happen. Today, our planet's path is elliptical in shape, with a value of 0.016. Therefore, the difference of seasonal insolation between summer and winter reaches to about 6%. So, this shows that the amount of solar radiation reaching to the Earth's surface is greater at perihelion. If the Earth's orbital eccentricity is at its 225 maximum value (0.07), namely the most elliptical shape, difference of seasonal insolation between summer and winter would reach to about 30%. As a result, seasons would be at their extremes in terms of temperature (e.g. very hot summers) for one hemisphere and would be moderate (e.g. milder summers) for the other hemisphere.

The second component of orbital variations is called axial tilt or obliquity. Our planet's rotational axis is tilted relative to its 230 orbital plane. The angle of the tilt is determined by drawing a line perpendicular to the Earth's orbital plane. The angle of the tilt changes between 21.5° to 24.5° on a 41000 year cycle (Fig. 4).





**Figure 4: The figure shows the maximum, minimum and present tilt angles for the Earth. Reprinted from Essentials of Geology (p. 283) by Lutgens, F.K., Tarbuck, E. J. and Tasa, D., 2012, U.S.A.: Prentice Hall. Copyright ©2012, 2009, 2006 by Pearson Education, Inc.**

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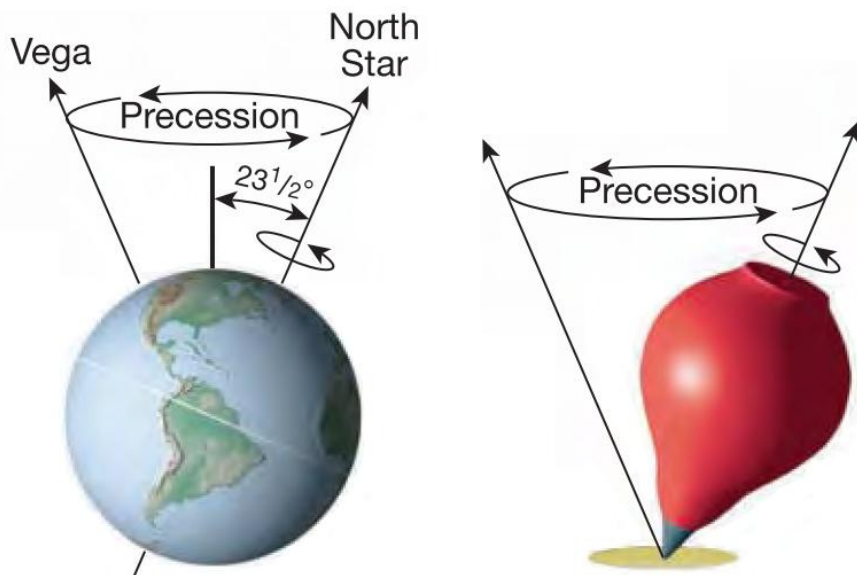
Today, the obliquity of earth is measured as  $23.5^\circ$ . This tilt can be measured easily at solstices and equinoxes. In order to do that, it is sufficient to take the inverse tangent of the value which is found by dividing an object's height by its shadow length, at that particular time.

240

When the angle of tilt is about to increase its maximum value  $24.5^\circ$ , the temperature contrast grow sharper between the two seasons. In such a case, winters become colder and summers become warmer. Contrarily, when the angle of tilt decreases, the characteristics of seasons come closer. In this case, milder winters follow cooler summers.

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The third and last component of orbital variations is called precession of the equinoxes. While orbiting the Sun, our planet also wobbles on its axis, like a spinning top. This wobbling of the Earth on its axis periodically repeats itself every 26.000 years. Today, the polar axis of the Earth points to Polaris, also known as the North Star. In fact, this won't last forever. Due to wobbling motion, the Earth's axis will gradually change and it will point to Vega. After 13000 years, Polaris will once again be the North Star (Fig. 5).



**Figure 5: The figure shows the wobbling motion of the Earth. Reprinted from Essentials of Geology (p. 283) by Lutgens, F.K., Tarbuck, E. J. and Tasa, D., 2012, U.S.A.: Prentice Hall. Copyright ©2012, 2009, 2006 by Pearson Education, Inc.**

250 The main effect produced by precession is an alternation of the seasons for each hemisphere. For example, today, the northern hemisphere is in winter, the southern hemisphere is in summer at perihelion. An opposite situation will be observed by the middle of the cyclic period. To put it differently, when the Earth's rotational axis points to Vega, the northern hemisphere would be in summer and the southern hemisphere would be in winter. In such a situation, the summers become warmer and the winters become colder. In other words, seasonal contrast in temperature appears stark.

255 While the information of orbital cyclicity was not new at all for many astronomers, there was no available calculation of past orbital variations until Urbain Le Verrier (1811–1877), the French astronomer and mathematician. By applying Newtonian laws to the masses of planets, he calculated the past changes of orbital motions of the Earth. In this way, he created a sort of data table that displays the past variations of Earth's orbit in the last 100000 years. After decades, U.S. American astronomer John Nelson Stockwell (1832–1920) calculated the changes in obliquity and eccentricity of eight planets in the solar system.

260 Furthermore, in 1904, German mathematician Ludwig Pilgrim (1844–1927) extended the calculations further and provided data on orbital changes of the Earth for the last million years.

Having these data is a big step forward for Milankovitch because they will be used in revealing the relationship, if there is any, between insolation amount and global climate change of the Earth. However, Milankovitch was extremely careful and meticulous in his endeavors. For this reason, despite having all the necessary information provided by Le Verrier, Stockwell  
265 and Pilgrim, he once more calculated the past variations of the Earth by inserting "more accurate values for the masses of the

planets in the solar system” (Grubic, 2006, p.199). After settling these preliminary points, the remaining task was essentially to combine the orbital variations of our planet with a mathematical model.

Milankovitch was well aware of the past astronomical theories and their shortcomings in explaining the phenomenon of glaciation. For example, Adhémar correctly emphasized 26.000 years cycle of precession as one of the most responsible  
270 factor for glaciation but considered the orbital eccentricity as a constant. Hence, this incorrect thought, no doubt, led him to conclude that ice ages could occur only in one hemisphere. In other words, his theory suggested that one hemisphere would experience ice age condition while the other would be ice-free during a period of 13.000 years. Croll did not make the same mistake and paid much attention to the orbital eccentricity of Earth. Furthermore, he was the first who noticed the idea of albedo, or the reflectiveness of the Earth’s surface. Although the effect of albedo is decisive to some extent, unfortunately  
275 that led him to think that colder winters would influence the expansion of glaciers. Thus, apart from neglecting the effect of obliquity, he insisted that ice ages are driven by very cold winters at aphelion.

According to Milankovitch, both earlier theories were incomplete because they do not contain all necessary parameters. In other words, either one or two orbital parameters are used in these early theories to give an explanation for glaciation. However, all parameters have an effect on the glaciation, one way or another and should be taken in account. Besides, there  
280 was one other consideration which is in one sense related to the first. According to Milankovitch, both early approaches were mathematically incapable of linking the effects of orbital variations on insolation. He states in his book, *Kanon der Erdbestrahlung und seine Anwendung auf das Eiszeitenproblem*, as follows:

All these theories have [...] the same shortcoming: None of them have correctly grasped the variability of all the astronomical elements which affect the irradiation of the Earth. [...] Besides, none of these theories  
285 was able to tackle mathematically the decisive influence of variations in the obliquity on the irradiation of the Earth (Milankovitch, 1941, p.376).

And then continues as follows:

I was able to show that the astronomical problem of ice ages is far more complicated than had been assumed before, and that in order to arrive at a correct solution the whole problem had to be approached  
290 fundamentally and put on a broad basis (Milankovitch, 1941, p.376).

Milankovitch’s fundamental approach was to base his work on a mathematical model. Before constructing the model, he set some preliminary ground for two concepts (see Grubic, 2006, p. 199). First, he defined the unit of radiation. He took “the solar constant as unit of radiation and a hundred thousandth part of the year as the time unit” (Milankovitch 1941, p. 266). Compared with the langley unit (1 langley unit = 1 gram calory/cm<sup>2</sup>), this unit is equivalent to .0576 langleys/day (see Lamb,  
295 2011, p.315). Second, he introduced the concept of caloric half-year. According to Milankovitch, the astronomical calendar was inappropriate to follow the secular march of insolation precisely because its length varies due to the variations of



astronomical elements. To avoid this problem, he divided a year into two halves as caloric summer half-year and caloric winter half-year. Like in the astronomical calendar, each year's duration is 182 days, 14 hours and 54 minutes, but differently a caloric winter includes every day which is colder than the days in the summer half year (see Milankovitch, 1941, pp. 271–275).

After having settled all these matters to his satisfaction, Milankovitch presented his mathematical model for insolation. The essential elements for the model are as follows: summer and winter caloric half-years for a certain year ( $\Delta Q_s$  and  $\Delta Q_w$ ), changes of irradiation at a certain latitude for summer and winter ( $\Delta W_s$  and  $\Delta W_w$ ), the change of the inclination of the ecliptic ( $\Delta \varepsilon$ ), eccentricity of the ecliptic in the given year ( $e$ ), longitude of the perihelion relative to equinox ( $\varpi$ ) and the coefficient ( $m$ ) for individual latitudes.

So, the model comprises all the necessary components including three orbital parameters, the changes of irradiation amount for specific latitudes and the coefficient that expresses the insolation amount in caloric units. With all these, the mathematical model which allows us to calculate the secular march of insolation for the northern hemisphere is as follows:

$$\Delta Q_s = \Delta W_s \Delta \varepsilon - m \Delta(e \sin \varpi) \quad (2)$$

$$\Delta Q_w = \Delta W_w \Delta \varepsilon + m \Delta(e \sin \varpi) \quad (3)$$

For the southern hemisphere, it takes the form as follows:

$$\Delta Q_s^- = \Delta W_s \Delta \varepsilon + m \Delta(e \sin \varpi) \quad (4)$$

$$\Delta Q_w^- = \Delta W_w \Delta \varepsilon - m \Delta(e \sin \varpi) \quad (5)$$

The numerical values of the elements in the model can be taken from all the relevant tables in the *Kanon*. Based on the Le Verrier, Stockwell and Pilgrim calculations, Table IX displays the changes in orbital eccentricity, axial tilt and precession; Table XII displays the changes of the irradiation amount for a particular latitude in summer half-year and winter half-year; and Table XIV contains numerical values of coefficients. According to Milankovitch “tables IX, XII, and XIV contain all data necessary to compute [...] the secular march of irradiation received by individual latitudes for the past six hundred thousand years” (Milankovitch, 1941, p.286).

In order to be a significant global change in the climate, all the cyclic variations of the orbital motions must be somewhat in a superposition. It is because all three cycles operates independently of each other. “Sometimes their influence on the amount of heat received by certain parts of the Earth nullifies each other. Sometimes the changes increase or decrease the quantity of the heat” (Grubic, 2006, p. 199). When the quantity of the heat decreases, a glacial period begins. In the opposite situation, when the quantity of heat increases, the global temperature significantly rises up and an interglacial period begins consequently.

With the Milankovitch model of insolation, it became possible to calculate how much solar radiation is received for any latitude in the past, present and future. Nevertheless, the model as such was not sufficient to understand the ice ages. To understand, completely, how the mechanism of ice age works, Milankovitch needed to know which seasons are crucial in



triggering the glaciation process. The early theories were centered on very cold winters and accepted such severe winters as a  
330 decisive factor in accelerating glaciers (see Imbrie and Imbrie, 1979, p.104). Milankovitch was skeptical about this  
hypothesis. However, being untrained in geological dynamics, he was far from abandoning this idea and putting an  
alternative approach. Yet, he was aware that he “had to solve” this “preliminary question of principal importance: which of  
the meteorological elements and which season was to be selected in the Ice–Period” (Milankovitch, 1941, p. 414).  
Eventually, the expected help came from the Russian–German geographer, and climatologist Wladimir Peter Köppen (1846–  
335 1940). He was aware of the works of Milankovitch and had read the monograph which is about the model of insolation.  
Soon after, he contacted Milankovitch. At the time, Köppen was researching on climate classification and climates of  
geological past with his son–in–law Wegener. Both researchers were experienced in the field, and they were familiar with  
almost all the relevant geological record as well as the seminal studies on ice ages. On a weekly basis, Köppen and  
Milankovitch exchanged dozens of mail. Finally, the discussions ended up, when Köppen convinced him that the decisive  
340 factor on glaciation was milder summers, not colder winters as usually thought. That did seem reasonable to Milankovitch.

After an exhaustive discussion off all the possibilities, Köppen answered the question by indicating that it  
is the diminution of heat during the summer half–year which is the decisive factor in glaciation... I  
therefore used Köppen’s advice and directed my attention to the periods of cold summers (Milankovitch,  
1941, p.414).

345 Truly, the suggestion to focus on milder summers was genuine because the key factor in the development of glaciation was  
about whether or not glaciers are preserved. At the high latitudes of the Northern Hemisphere, large amount of snow were  
accumulating and the glaciers were being formed even in completely different past climatic conditions. However, when the  
summers were hotter in any hemisphere, the glaciers formed in the winter were starting to melt slightly. Therefore, while  
milder or relatively colder summers were preserving glaciers and hence culminating glaciation; hotter summers were setting  
350 the stage for interglacial periods (see also Oerlemans, 1991).

Milankovitch had already constructed a model that makes it possible to calculate the insolation amount for a particular given  
latitude of the Earth. However, caloric half–years and canonic units were still not included. This model was constructed on  
the basis of astronomical half–years:

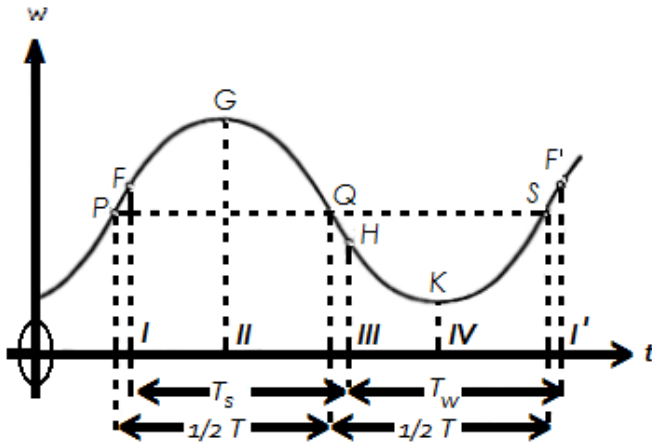
$$W_s = W_s^0 + \Delta W_s \cdot \Delta \varepsilon \quad (6)$$

355  $W_w = W_w^0 + \Delta W_w \cdot \Delta \varepsilon \quad (7)$

where  $\varepsilon_0$  denotes the present,  $\varepsilon$  refers to a particular past value of obliquity and  $\Delta \varepsilon = \varepsilon - \varepsilon_0$ . The values of all other elements in  
the model ( $W_s^0$  and  $W_w^0, \Delta W_s$  and  $\Delta W_w, \Delta \varepsilon$ ) can be found in the Tables XI, XII and IX, in the *Kanon*, respectively. With  
this model, the insolation amount of any given latitude ( $\varphi$ ) can be calculated for both summer and winter half–years. In this  
early version of the model, the strategy was to determine the difference between present and past values of insolation. If any



360 difference appears between the past and present values of astronomical elements, it could be represented with a graph (Fig.  
 6).



**Figure 6: The annual march of the irradiation for any arbitrary latitude. Reprinted from *Kanon der Erdbestrahlung und seine Anwendung auf das Eiszeitenproblem* (p.272) by Milankovitch, M., 1941, Belgrade: Royal Serbian Academy Special Publications.**

365 In the graph, we have the representation of the mean radiation  $w$  of a surface element at any arbitrary latitude ( $\varphi$ ) as a  
 function of time. The points on the curve PFGHKSF' represents the course of radiation at the latitude ( $\varphi$ ). The points I,II, III,  
 IV and I' on the timeline, respectively denotes the time of the vernal equinox, the summer solstice, the autumnal equinox, the  
 winter solstice and the subsequent equinox. The caloric half-years, as stated above, have not been introduced here yet, hence  
 the astronomical summer half-year ( $T_s$ ) is represented by the segment I–III, and the astronomical winter half-year ( $T_w$ ) is  
 370 represented by the segment III–I' (see Milankovitch 1941, 271–294).

The model was incomplete to some extent, but it was useful for calculating the insolation amount of any given latitude. For  
 all that, Milankovitch took the opportunity to revise the model before his calculations. In this new version of the model, the  
 strategy was nearly as same as the first; comparing the values of the past astronomical parameters with the present ones.  
 Additionally, this time, caloric half-years and canonic units were included in the model. As a result, the insolation amounts  
 375 of any latitude were ready to be calculated with the following model:

$$Q_s = W_s^0 + \Delta W_s \Delta \varepsilon \mp m \varepsilon \quad (8)$$

$$Q_w = W_w^0 + \Delta W_w \Delta \varepsilon \pm m \varepsilon \quad (9)$$

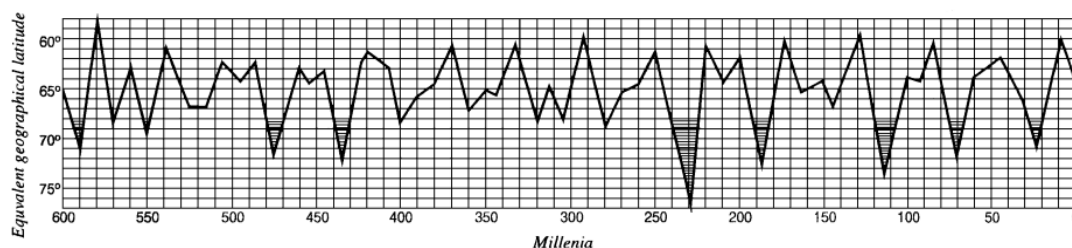
Here, both the caloric half-years and canonic units are included in this revised version of the model. Additionally, the most  
 important things that should be considered are upper and lower signs of both equations. According to Milankovitch, when  
 the longitude of the perihelion is  $90^\circ$  the upper sign is valid and when it is  $270^\circ$  the below sign is valid. This is because  
 380 extreme changes in the insolation amount of considered latitudes (which are thought by Köppen to be 55th, 60th and 65th





parallels of the northern hemisphere) were happening when the degrees are at  $90^\circ$  and  $270^\circ$  with respect to the vernal equinox.

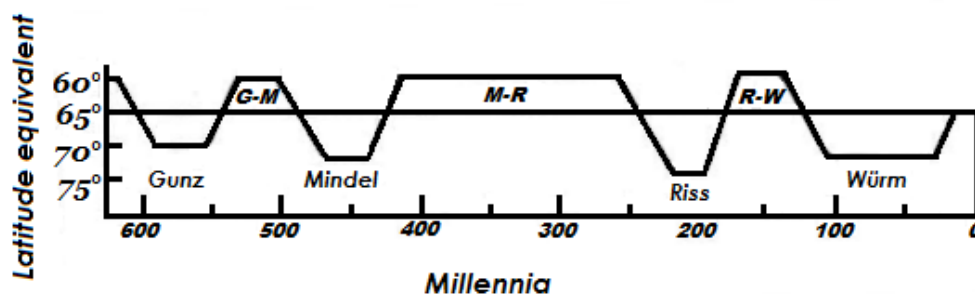
Using the latest version of his model, Milankovitch calculated past insolation amounts for the specific latitudes. He obtained the intended results over months and transformed these into fictitious latitudinal oscillations. The reason for that was to see a geometrical picture of the past summer insolation variations for the northern latitude of  $65^\circ$ . The created graph was an equivalent graph which showed the equivalent values of summer insolation amount for  $65^\circ\text{N}$  throughout the past 600.000 years (Fig. 7).



390 **Figure 7: The equivalent graph which shows the variations of the summer insolation for  $65^\circ\text{N}$ . Reprinted from *Kanon der Erdbestrahlung und seine Anwendung auf das Eiszeitenproblem* (p.415) by Milankovitch, M., 1941, Belgrade: Royal Serbian Academy Special Publications.**

As seen in the graph, the past equivalent insolation values of  $65^\circ\text{N}$  are shown by curves for comparison. For example, about 115.000 years ago, the summer insolation amount of  $65^\circ\text{N}$  was almost equal to the today's summer insolation amount of  $74^\circ\text{N}$ . This could be evaluated as an indication of a relatively cold summer. In the opposite cases, the same latitude would of course have relatively warmer temperature averages in the summers.

Soon after completing his work, Milankovitch sent this radiation curve graph to Köppen. The graph must have been intriguing for Köppen, as it was in agreement with the findings of two German geographers Albrecht Penck (1858–1945) and Eduard Brückner (1862–1927). Both scientists were already presented their results in a graph (Fig. 8).



400 **Figure 8: Penck and Brückner's scheme of glacial/interglacial periods in the Alpines. Reprinted from *Kanon der Erdbestrahlung und seine Anwendung auf das Eiszeitenproblem* (p.417) by Milankovitch, M., 1941, Belgrade: Royal Serbian Academy Special Publications.**



Penck and Brückner were researching on Alpine glaciers and about 15 years before Milankovitch's work, they identified  
405 four great glacial periods in Earth's history by examining successive gravels, plant remains and moraines in the European  
Alps (Anderson *et al.*, 2013 p. 8). They displayed these different periods in a graph and named them chronologically as  
Günz, Mindel, Riss, and Würm. The close agreement of the two graphs was a sign of victory for Milankovitch. Moreover,  
with this new graph, Milankovitch provided a more precise graph compared to the old one. As seen in (Fig. 7), the nine  
striped low points of the curve display colder summers which correspond to glacial periods occurred in the 589<sup>th</sup>, 548<sup>th</sup>,  
410 475<sup>th</sup>, 434<sup>th</sup>, 231<sup>st</sup>, 187<sup>th</sup>, 116<sup>th</sup>, 72<sup>nd</sup>, and 22<sup>nd</sup> Millennia BP. Günz, Mindel and Riss, each includes two, and the last  
significant cold period Würm includes three low points. Clearly, Penck and Bruckner scheme does not have this preciseness.

## 7 Conclusion

On Milankovitch's part, the goal had been finally reached. His work "based upon exact science" passed into "the sphere of  
the descriptive natural sciences" and made the "link between celestial mechanics and geology" (Milankovitch, 1941, xvi). In  
415 short, exact science and descriptive science met by means of geological research on the one side and astronomical  
computation on the other. Furthermore, by constructing a mathematical model based on the orbital variations of Earth, it  
became possible to trace past climatic variations and also to predict the future glacial or interglacial periods. For example,  
the model predicts that, with or without human effects, interglacial period which we are still in may end about 50.000 years  
after (see Berger and Loutre 2002; Gajic 2019).

420 Although, in 1924, Köppen and Wegener confidently published radiation curves in their book *Die Klimate der geologischen  
Vorzeit*, some remained sceptical about the validity of Milankovitch's model until 1970's. In the following years, however,  
Hays *et al.* (1976) published an article on the close link between climatic changes and the Earth's orbital variations. Based  
on the oxygen isotope records provided by deep ocean sediment cores, they came to the conclusion that orbital changes  
induced climatic change in the past 500.000 years. In their terms, orbital cycles of the Earth should be understood as  
425 pacemakers of the ice ages. Eventually, this could be counted as a clinching victory for Milankovitch's model. In other  
words, Milankovitch's work was not merely providing a basis for further studies on mathematical climatology; it was also  
reliable in estimating the relationship between orbital variations and climatic changes, in a robust manner.

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