History of Geo- and Space Sciences

The length of coastlines in Ptolemy's *Geography* and in ancient *periploi*

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Abstract. The lengths of the coastlines in Ptolemy's *Geography* are compared with the corresponding values transmitted by other ancient sources, presumably based on some lost *periploi* (literally "voyages around or circumnavigations", a genre of ancient geographical literature describing coastal itineraries). The comparison reveals a remarkable agreement between them, suggesting that Ptolemy relied much more heavily on these or similar *periploi* than it used to be thought. Additionally, a possible impact of Ptolemy's erroneous estimate of the circumference of the Earth is investigated. It is argued that this error resulted in two interrelated distortions of the coastal outlines in Ptolemy's *Geography*. First, the north–south stretches of the coast that were tied to particular latitudes are shown compressed relative to the distances recorded in other sources in roughly the same proportion to which Ptolemy's circumference of the Earth is underestimated relative to the true value. Second, in several cases this compression is compensated by a proportional stretching of the adjacent east–west coastal segments. In particular, these findings suggest a simple explanation for the strange shape of the Caspian Sea in Ptolemy's *Geography*.

1 Introduction

Classics (or studies of Greco-Roman antiquity), which is the field that the history of ancient geography and cartography belongs to, is a very old discipline. So, as time goes by, the chance to find something really new in this field, or to do something that nobody has ever done before, tends to zero. Ptolemy's *Geography* (ca. AD 150) is a happy exception to this tendency. Although the *Geography* is one of the most famous and often-studied works in the history of cartography, its origins still remain an enigma. However, in the last decade, after the publication of its new edition by Stückelberger and Graßhoff (2006), Ptolemy's *Geography* has seen an upsurge of scholarly interest. Numerous researchers have developed a wide array of different methodological approaches to studying its nature and genesis (e.g. Isaksen, 2011, 2013; Marx, 2011, 2016; Heß, 2016; Shcheglov, 2017; Graßhoff et al., 2017; Arnaud, 2017; Defaux, 2017). One of the main approaches is a comparison with other ancient sources. Surprisingly, until this day nobody has ever tried to compare Ptolemy's *Geography* with the ample data on coastline length provided mostly by Agath-

emerus in his *Geographical Sketch* (the 1st century AD) and by Pliny the Elder in the geographical books of his *Natural History* (AD 77–79). The objective of the present study is to fill this gap. My contention is that this comparison can shed some new light on Ptolemy's *modus operandi* and the genesis of his *Geography*.

2 *Periploi* among the sources of Ptolemy's *Geography*

Ptolemy's *Geographical Guide*, or simply the *Geography*, was essentially a description of the world map in terms of spherical coordinates (latitude and longitude in degrees) assigned to each of the 6300 listed localities.[1] However, it is clear that only some of these coordinates could have been based on astronomical observations (e.g. the latitudes of the

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[1]Consequently, the term "Ptolemy's *Geography*" may be used synonymously with "Ptolemy's map" when referring to a map reconstructed from the coordinates recorded in the text. These coordinates are now available in an electronic database attached to the edition of the *Geography* by Stückelberger and Graßhoff (2006).

major cities: Alexandria, Rhodes, Athens, Rome, Massalia, etc.). Ptolemy's predecessors and possible sources described geographic space mostly in terms of distances measured in customary units (Greek stades, Roman miles, etc.).[2] It is reasonable to suppose, therefore, that, with few exceptions, Ptolemy's coordinates were derived from distances converted to angular units (Cuntz, 1923, p. 110; Knapp, 1996, p. 30–35; Graßhoff et al., 2017; Arnaud, 2017, pp. 3, 14–20). This makes comparison of Ptolemy's *Geography* with other surviving sources a promising way to investigate its organization and genesis. Interestingly, there are a lot of studies comparing Ptolemy's distance data with the Roman itineraries (or descriptions of the land routes),[3] but only a few attempts to compare them with the Greek *periploi*.[4]

Periploi (sing. *periplus*) is a genre of ancient geographical literature describing coastlines in the form of a linear succession of stopping points and intervening distances. *Periploi* are considered to be "the standard basis of ancient descriptive geography" (Purcell, 2012, p. 1107).[5] Many ancient geographers – Artemidorus, Strabo, Pomponius Mela, Pliny the Elder (in Books III–VI of his *Natural History*), Dionysius Periegetes, etc. – presented their works in the form of *periploi*, even though they actually did not confine their narratives to the coasts. Ptolemy himself emphasizes (*Geogr.* 1.18.6) that it is much easier to indicate the positions of the coastal cities than of the inland ones. It is reasonable to suppose, therefore, that in many respects it was *periploi* that provided the foundation for his map of the world.

Only a few *periploi* survived from antiquity and only four of them provide detailed information on distances: three *Periploi* of the *Pontus Euxinus* (Black Sea) and the *Stadiasmus of the Great Sea* (Mediterranean).[6] These sources, however, deserve a more thorough examination elsewhere. Fortunately, thanks mainly to the *Geographical Sketch* by a certain Agathemerus (Diller, 1975) and to Books III–VI of Pliny's *Natural History* (Jan and Mayhoff, 1892), as well as to some other sources, we have a set of values for the total coastline lengths of the major regions as recorded by the most famous ancient geographers: Eratosthenes (the 3rd quarter of the 3rd century BC),[7] Artemidorus of Ephesus (ca. 104–100 BC),[8] Marcus Vipsanius Agrippa (ca. 12 BC),[9] Isidore of Charax (around the turn of the 1st centuries BC and AD), and some others. These values evidently derive from some lost *periploi*.

This data set covers the largest part of the then-known world (Figs. 2 and 3). First of all, Artemidorus, Isidore, and Agrippa give the lengths of the coastlines of the "Inner" Sea (Mediterranean, Black, and Azov seas) as a whole and as divided between the three continents: Europe from the Pillars of Hercules (or Calpe, i.e. Gibraltar) to the mouth of the Tanais (Don), Asia from the Tanais to the Canobic (westernmost) mouth of the Nile, and Libya (Africa) from this mouth to Tingis (Tangier). The same geographers and some other authors give the perimeters of the major peninsulas (the Pyrenean, the Italian, the Arabian), seas or gulfs (the Persian Gulf and the so-called "second" and "third" gulfs of Europe i.e. roughly speaking, the Adriatic, Ionic, and Aegean seas and the Red, Black, Azov, and Caspian seas), and the largest islands. The fact that different authors reproduce almost the same or similar set of values suggests that these data were regarded as a special category to be included in a geographical treatise. For the sake of completeness, we will add to this set several values of the total coastline lengths reported by the surviving "detailed" *periploi*: Pseudo-Arrian's *Periplus* and the *Stadiasmus*, as well as by Strabo's *Geography* (ca. AD 23), our main source on its subject matter (Radt, 2003–2011).

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[2]The length of the stade is a vexed issue. The majority of ancient sources use the ratio of 1 Roman mile (for which the standard notation is m.p. or *mille passuum*) to 8 Greek stades. This ratio is used throughout the present paper. Since the Roman mile measures about 1480 m, 1 stade is equal to ca. 185 m. For a detailed discussion with references to relevant literature, see Shcheglov (2016b, p. 694–701).

[3]Cuntz (1923), Spaul (1958, p. 5–7), Seabra Lopes (1995–1997), Knapp (1996), Meuret (1998), Gómez Fraile (2005), Urueña Alonso (2014a, b).

[4]Mittenhuber (2012) compares Ptolemy's Hispania (i.e. the Iberian Peninsula) with the distance data from the so-called Artemidorus papyrus, while Marx (2016) compares Ptolemy's Atlantic coast of Africa with the distance data from Pliny's *Natural History*.

[5]On *periploi* as one of the basic forms of geographical description in antiquity, see Janni (1984).

[6]Two *Periploi of the Pontus Euxinus* (Black Sea) composed by Menippus of Pergamon (the end of the 1st century BC) and Flavius Arrian (the beginning of 2nd century AD) served as the sources for the third and fuller *Periplus* known under the name of Pseudo-Arrian (3rd century AD). The standard edition of these three *periploi* is Diller (1952, p. 103–146). The anonymous *Stadiasmus of the Great Sea* (1st century AD; for this dating, see Uggeri, 1996) describes the coasts of Africa from Alexandria to Carthage and of Asia

from Aradus to Miletus including the adjacent islands of Cyprus and Crete. The most reliable edition is still Müller (1855, p. 427–563); a new edition is being prepared by Pascal Arnaud. These *periploi* may be called "detailed" in the sense that, unlike other surviving *periploi*, they divide the route into hundreds of course legs with the median length of ca. 90 stades or 16–17 km. On the ancient methods of maritime distance estimation, see Arnaud (1993).

[7]Eratosthenes was probably the most famous ancient geographer before Ptolemy. His geographical treatise has survived only in fragments; for the editions, see Berger (1880) and Roller (2010).

[8]Schiano (2010).

[9]Marcus Vipsanius Agrippa (63–12 BC) was the right arm of the emperor Augustus. According to Pliny's testimony (*Natural History* 3.17), he left behind an unfinished geographical work which, as most scholars believe, served as the basis for a world map displayed in the Porticus Vipsania in the centre of Rome (before AD 14). The map was accompanied by explanatory notes with an account of dimensions and boundaries of 24 regions of the world. These notes have come down to us only in fragments; the standard edition is Klotz (1931).

The objective of this paper is to compare this data set with the lengths of the corresponding coastlines in Ptolemy's *Geography*. But before attempting the comparison there are several methodological issues we need to discuss.

3　Method of comparison

How can we measure the lengths of the coastlines in Ptolemy's work? My contention is that the simplest and most natural way would be to take Ptolemy's *Geography* as what it essentially is, namely as a catalogue of spherical coordinates. The distance between two points specified by these coordinates is an arc of the great circle that can be calculated using the rules of spherical trigonometry.[10] Accordingly, I propose to calculate the length of the coastlines in Ptolemy's *Geography* as the sum of individual arcs joining coastal points multiplied by the length of a degree of the great circle which he defines as 500 stades (*Geogr.* 1.7.1, 11.2).

It is important to emphasize that actually next to nothing is known about how Ptolemy worked with his sources and how he converted their distance data into coordinates (Graßhoff et al., 2017, p. 2, 6–7; Defaux, 2017, p. 12, 255). It is hard to imagine that he would have calculated the coordinates for each of the 6000 points listed in his *Geography*. It would be more reasonable to assume that calculations may have been performed only for the most important points, whereas all others were localized by means of simpler methods, for example, Pythagoras' theorem and/or a simple ruler; cf. similarly Spaul (1958, p. 5–7) and Graßhoff et al. (2017, p. 1, 16, 25).

Ptolemy's coordinates are specified with precision up to $5'$. This, however, must not mislead us about the accuracy of his data. The precision of Ptolemy's coordinates correlates clearly with the density of the points on his map: its "rarest" parts at the periphery (such as Asian Scythia or Inner Libya) have the highest percentage of coordinates in whole degrees, but as we approach the Mediterranean, the fractions of a degree become increasingly more frequent and smaller. In other words, Ptolemy's coordinates can be precise not so much because his sources were accurate, but rather in order to fit many points in a confined space. Statistical analysis of the frequency of different fractions of a degree used in Ptolemy's coordinates shows that he tended to round all values to the largest possible fractions.[11] It means that, fig-

uratively speaking, Ptolemy's map had "low resolution": it could reproduce general outlines of large objects, but was quite inaccurate in details. For this reason, when comparing Ptolemy's data with the distances reported by other sources, it makes sense to consider only those applying to relatively long intervals. It also needs to be born in mind that, owing to the peculiarities of the map, close matches can hardly be expected from this comparison.

The text of Ptolemy's *Geography* has been handed down to us in two recensions: Ξ and Ω (Fig. 1).[12] Both of them descend from antiquity, but none of the extant manuscripts dates earlier than the late 13th century. The Ξ recension is considered to be earlier and more authentic, but it is represented by the sole manuscript Vaticanus Graecus 191 (ca. AD 1295) which omits all coordinates for the eastern half of the map (i.e. for the whole Asia, except for Asia Minor, Armenia, and Asiatic Sarmatia) and contains quite a few scribal errors. The Ω recension is apparently secondary to Ξ, but it includes the majority of the manuscripts.[13] In view of these circumstances, when comparing Ptolemy's data with the distances recorded in other sources, both recensions should be taken into account, but Ξ should be regarded as more reliable.

4　Comparison of the coastline lengths in Ptolemy's *Geography* and in other sources

Let us turn to the sources. All data needed for comparison are given in Table 1, which constitutes the core of this paper, and are illustrated by Figs. 2–7. Table 1 compares the coastline lengths derived from Ptolemy's coordinates (in Ξ and Ω) with the corresponding distances given in the other sources, presumably based on *periploi*.[14] Also, the table shows how much Ptolemy's values deviate from those of the

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[10]The distance between points A and B situated on a sphere can be calculated by the formula $\cos S_{AB} = \cos \Delta\lambda_{AB} \cdot \sin(90° - \varphi_A) \cdot \sin(90° - \varphi_B) + \cos(90° - \varphi_A) \cdot \cos(90° - \varphi_B)$, where S_{AB} is the distance between points A and B expressed in degrees of the great circle, $\Delta\lambda_{AB}$ is the longitudinal interval between them, φ_A and φ_B are their latitudes. Certainly, Ptolemy did not have trigonometric formulas in their modern form, but the theorems of Menelaus that he used to solve similar problems in the *Almagest* were an ancient equivalent of them (Neugebauer, 1975, p. 21–30).

[11]The frequency of different fractions of a degree in Ptolemy's catalogue of coordinates conforms to a normal distribution: the co-

ordinates in integer degrees, without fractions, are the most frequent, those with $1/2°$ are less frequent, those with $1/3$ or $2/3°$ are still less, those with $1/4$ or $3/4°$ and $1/6$ or $5/6°$ are still less again, and those with $1/12$, $5/12$, $7/12$ or $11/12°$ are the rarest; see Wurm (1931, p. 25–27), Marx (2011, p. 30–36), Isaksen (2011, p. 262–264, 2013, p. 48–50).

[12]For a detailed discussion of the manuscript (MSS) tradition, see e.g. Schnabel (1938), Stückelberger and Graßhoff (2006, p. 30–34), Burri (2013, p. 63–93), for a good synopsis, see Defaux (2017, p. 67–81).

[13]See e.g. Cuntz (1923, p. 15–16), Schnabel (1938, p. 44, 55–57), Burri (2013, p. 540–542). Polaschek (1965, cols. 717, 742–744) and Berggren, Jones (2000), p. 43–45 argued that the Ω recension represents a post-Ptolemaic Byzantine revision, whereas codex Vaticanus Graecus 191 is the only piece of Ptolemy's genuine text. However, I see no sufficient grounds for such a radical position; see similarly Defaux (2017, p. 80–81).

[14]All Ptolemy's coordinates are taken from the electronic database attached to the newest edition of the *Geography* (Stückelberger and Graßhoff, 2006) and are appended to the present paper as Excel files in the Supplement.
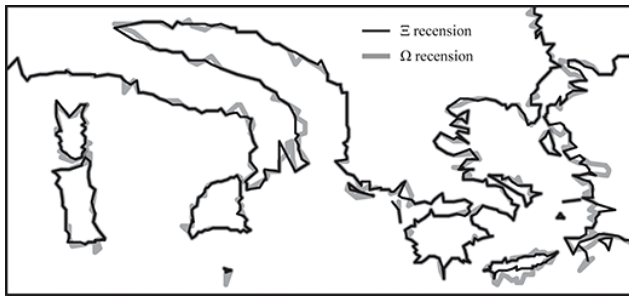
Figure 1. The outlines of the Mediterranean Sea according to the Ξ and Ω recensions of Ptolemy's *Geography* (Italy, the Balkans, and the Aegean Sea are taken as an example). This and all the other maps in the paper are drawn using the projection attributed by Ptolemy to Marinus of Tyre (*Geogr.* 1.20.3–5), whom he introduces as his immediate predecessor and the primary source of information (*Geogr.* 1.6.1). In modern terms, it is an equidistant cylindrical projection, in which the ratio between equal latitudinal and longitudinal intervals is represented as 5 : 4.

other sources, and how much Ω deviates from Ξ, both in terms of stades and in terms of percentage.

What conclusions can be drawn from this comparison? First of all, our sample clearly falls into two parts: shorter and longer distances. Islands and other small features (such as Peloponnesus or no. 18 in Table 1, the Great Syrtis or nos. 7 and 17, etc.) or relatively short coastal stretches exhibit little similarity between Ptolemy's data and the *periploi* data (the only exceptions are Sicily nos. 15 and 16, Crete no. 8, and Illyria nos. 1 and 9; Figs. 3 and 7) This result accords with the fact that Ptolemy's map had a low resolution and was in principle incapable of representing small features accurately.

However, for longer distances, with a threshold being approximately at 10 000 stades, the pattern changes radically. It is instructive to remember that, with only a few exceptions, these coastlines are very long and have very complex geometry. In these circumstances, one can hardly expect any similarity between Ptolemy's figures and the *periploi* figures at all. Nevertheless, the correspondences between them are so numerous and so close (Fig. 4) and their geographical coverage is so wide (Figs. 2 and 3) that this can reasonably be regarded as something more than just a series of coincidences.

Figure 4 shows that in most cases the differences between Ptolemy's data and the *periploi*-based sources fall between +4 and −1 %.[15] How can we interpret these values? Is it a lot or not? Previous researchers who compared Ptolemy's data with the distances recorded in other sources regarded

the 5–10 % difference between them as small enough to suggest a common origin (Cuntz, 1923, p. 120–122, 144–145; Urueña Alonso, 2014a, p. 164, 169–170; Marx, 2016, p. 33–34). But for a more robust assessment, it would be informative to compare the differences between Ptolemy and the other sources with the differences between the two recensions of Ptolemy's own work (see Table 1 and Figs. 5, 6). A convenient benchmark for this comparison is provided by the measurements of the "Inner" Sea (Mediterranean and Black seas without the Azov Sea).[16] In the Ξ recension, its perimeter measures 133 792 stades, which is 2.8 % longer than according to Artemidorus (130 120, no. 50 in Table 1), but in Ω it is 1.8 % longer (136 231) than in Ξ. Remarkably, in a number of cases the differences between Ξ and Ω are appreciably larger than those between Ξ and the other sources: Europe from Calpe to the Tanais (no. 48 in Table 1), Hispania (33), Italy (29), the "second gulf" of Europe (26), and the Red Sea (28). If we still believe that Ξ and Ω are, despite all the disagreements between them, two versions of the same work by the same author, the similarity between Ptolemy's Ξ values and those reported by the other sources cannot be dismissed as a coincidence.

It makes sense to take a closer look at the outliers. First of all, however obvious it may seem, it is worth noting that, when ancient sources give different estimates for the same coastline, Ptolemy's data cannot agree with all of them. For instance, when Ptolemy's data match one of the two known values for the perimeter of Arabia (no. 45: 38 120 stades), they must naturally disagree with the other (no. 44: 32 000). The same is true for the Great Syrtis (nos. 7 and 17), the Maeotis (22, 25), Hispania (33, 38), the Pontus Euxinus (35–37, 39), Africa (40–43), and the Mediterranean without the Pontus (49, 50). Taking this into account, only two outliers stand out: the Red Sea coast of Arabia (no. 27) and the Mediterranean coast of Africa (nos. 40–43). The estimated 14 000 stades for the Arabian coast, repeated by Eratosthenes, Artemidorus, and Agrippa, goes back as far as to Alexander's commander Anaxikrates and was regarded as "too much" (ἐπὶ πλέον) already by Strabo (16.4.4 C768). But it seems that the best way to explain a drastic reduction of this coast by 22 % in Ptolemy's *Geography* would be to link it to his adoption of an erroneous value for the circumference of the Earth (see below Sect. 5). The case of Africa seems to be more complicated: Ptolemy's distance is 5 % shorter than the *Stadiasmus*' value for the coast between Alexandria and Utica (no. 30), but it is 5.66 % longer than the largest value recorded by Pliny (no. 43) for the coast between Canobus and Tingis. Yet, it is remarkable that Pliny calls this value "the average of all the various accounts" (6.208: *ut media ex*

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[15]Of course, it is possible to find many contradictions between Ptolemy's data and distances reported by other sources, but there is no reason to expect them to match always and everywhere. For example, Pliny gives evidently incredible and fictional values for the coastlines of Germania (4.98 = Agrippa F 21; see Riese, 1878, p. 5) and Gaul (4.105 = Agrippa F 23 Riese = F 40 Klotz): 20 000 and 14 000 stades, respectively.

[16]It is important to emphasize that Artemidorus, followed by the majority of other ancient authors, estimated the circumference of the Maeotis (Azov Sea) at 9000 stades, whereas Ptolemy may have adopted an alternative estimate of 11 248 stades attested by Pliny 4.78.

Table 1. Comparison of the coastline lengths in Ptolemy's *Geography* and in the other sources.

No.	Coastline of	Ancient geographer	*Periploi*-based sources Source	Distance (D)	Ptolemy Distance Ξ	Ptolemy Distance Ω	Deviation Ξ (Ξ–D) in stades	Deviation Ξ (Ξ–D)/D×100 in %	Deviation Ω (Ω–D) in stades	Deviation Ω (Ω–D)/D×100 in %	Difference Ξ and Ω in stades (Ω–Ξ)	Difference Ξ and Ω in % (Ω–Ξ)/Ξ×100
1	Illyria from the Drinum (Drilon) to the Acroceraunia	Agrippa	F 13 Riese = F 16 Klotz = Plin. 3.150	1400	1350	1388	–50	–3.6	–12	–0.9	38	2.8
2	Corsica (perimeter)	Pliny	3.81	2600	4060	4283	1460	56.2	1683	64.7	223	5.5
3	Cyprus (perimeter)	Isidore	Plin. 5.129	3000		4204			1204	40.1		
4	Cyprus (perimeter)	Stadiasmus Maris Magni	315	3250		4204			954	29.4		
5	Caspian Sea from the Casus to the Cyrus[a]	Agrippa	F 31 Riese = F 53 Klotz = Plin. 6.39	3400	3726	3728	326	9.6	328	9.6	2	0
6	Cyprus (perimeter)	Timosthenes, Artemidorus	Strab. 14.6.2 C682; Agathem. 4.26. Plin. 5.129	3420		4204			784	22.9		
7	Great Syrtis from Cape Cephalae to Northern Cape	Strabo	17.3.20 C835	3930	3544	3548	–386	–9.8	–382	–9.7	4	0.1
8	Crete (perimeter)	Agathemerus	4.26	4100	3909	4556	–191	–4.7	456	11.1	647	16.5
9	Illyria from the Arsia to the Drinum (Drilon)	Agrippa	F 13 Riese = F 16 Klotz = Plin. 3.150	4240	4212	4431	–28	–0.7	191	4.5	219	5.2
10	Sicily (perimeter)[b]	Posidonius	F 249 Edelstein-Kidd = Strabo 6.2.1 C266	4400	4714	4968	314	7.1	568	12.9	254	5.4
11	Peloponnesus (perimeter)	Isidore	Plin. 4.9	4503	6436	6591	1933	42.9	2088	46.4	156	2.4
12	Sardinia (perimeter)	Pliny	3.84	4520	5054	5953	534	11.8	1433	31.7	900	17.8
13	Crete (perimeter)	Stadiasmus		4537	3909	4556	–628	–13.8	19	0.4	647	16.5
14	Crete (perimeter)	Pliny	4.58	4712	3909	4556	–803	–17	–156	–3.3	647	16.5
15	Sicily (perimeter)	Timosthenes	Agathem. 5.20	4740	4714	4968	–26	–0.6	228	4.8	254	5.4
16	Sicily (perimeter)	Agrippa	F 7 Riese = F 13 Klotz = Plin. 3.86	4944	4714	4968	–230	–4.7	24	0.5	254	5.4
17	Great Syrtis from Cape Cephalae to Northern Cape	Eratosthenes	Agathem. 3.8	5000	3544	3548	–1456	–29.1	–1452	–29.0	4	0.1
18	Peloponnesus (perimeter)	Artemidorus	Agathem. 4.24	5627	6436	6591	809	14.4	964	17.1	156	2.4
19	Italy, the southern side from the Varus to Rhegium	Pliny (Varro ?)	3.49, 51, 56, 62, 70, 73	7120	7387	8262	267	3.7	1142	16	875	11.8
20	Hispania from Gades to the harbour of the Artabrians[c]	Artemidorus	Agathem. 4.16	7932	6756	7075	–1176	–14.8	–857	–10.8	319	4.7
21	Asia from Balaneae to Caunus	Stadiasmus	the sum of all sections of the route	8240	6754	6577	–1486	–18	–1663	–20.2	–177	–2.6
22	Lake Maeotis (perimeter without the strait)	Artemidorus	Agathem. 3.10; Strab. 2.5.23 C126; 7.4.5 C310; Plin. 4.78; 6.207; Arrian, Periplus 30; Ps.-Arrian, Periplus 72(43). 121(92)	9000	11430	11249	2430	27	2249	25	–182	–1.6
			Distances over 10 000 stades									
23	Persian Gulf from Harmozon to the eastern mouth of the Tigris	Eratosthenes	F 94 = Strab. 16.3.2 C766	10000		10368			368	3.7		
24	Persian Gulf from Cape Makes (Asabon) to the eastern mouth of the Tigris	Eratosthenes	F 94 = Strab. 16.3.2 C767	10000		10412			412	4.1		
25	Lake Maeotis (perimeter without the strait)	Pliny	4.78	11248	11430[d]	11249	182	1.6	1	0	–182	–1.6
26	"Second gulf" of Europe from Lacinium to the Acroceraunia[e]	Varro (?) or Agrippa (?)	Plin. 3.97, 150	13600	13814	14607	214	1.6	1007	7.4	793	5.7
27	Red Sea coast of Arabia	Eratosthenes, Artemidorus, Agrippa	Erat. F 95 = Strab. 16.4.4 C768; Agathem. 3.14; Agrippa F 34 Riese = Plin. 6.164	14000		10882			–3118	–22.3		
28	Red Sea from Heroonpolis to Deire	Artemidorus	Agathem. 3.13	15500	15519	16074	19	0.1	574	3.7	555	3.6

Table 1. Continued.

No.	Coastline of	Ancient geographer	Source	Distance (D)	Ptolemy Distance Ξ	Ptolemy Distance Ω	Ξ−D in stades	(Ξ−D)/D×100 in %	Ω−D in stades	(Ω−D)/D×100 in %	Ω−Ξ in stades	(Ω−Ξ)/Ξ×100 in %
29	Italy between the mouths of the Varus and the Arsia	Pliny	3.44	16392	16763	18111	371	2.3	1719	10.5	1348	8
30	Africa from Alexandria to Utica	Stadiasmus Maris Magni, Eratosthenes,	the sum of all sections of the route; Agathem. 3.12–13; Erat. F 93	17304	16447	16911	−857	−5	−393	−2.3	464	2.8
31	Persian Gulf (perimeter)	Artemidorus	Roller = Plin. 6.108	20000	20780				780	3.9		
32	Caspian Sea (perimeter)[g]	Artemidorus	Agathem. 3.12–13 Plin. 6.37 ;	20000	20350	20390	350	1.7	390	2	41	0.2
33	Hispania (perimeter)[h]	Pliny (Varro ?)	4.118	20800	20610	21246	−190	−0.9	446	2.1	635	3.1
34	Asia from Canobus to Chalcedon[i]	Timosthenes	Plin. 5.47	21104	21977	22092	873	4.1	988	4.7	115	0.5
35	Pontus Euxinus (perimeter without the straits)[k]	Eratosthenes	F 116 Roller = Plin. 5.47; F 115 Roller = Plin. 6.3	23068	23551	23832	483	2.1	764	3.3	282	1.2
36	Pontus Euxinus (perimeter including the straits)	Strabo	the sum of all sections of the route	23290	23858	24123	568	2.4	833	3.6	265	1.1
37	Pontus Euxinus (perimeter including the straits)	Artemidorus	Plin. 4.77	23352	23858	24123	506	2.2	771	3.3	265	1.1
38	Hispania (perimeter)	Pliny (Agrippa ?)	4.118 = Agrippa F 39 Klotz	23392	20610	21246	−2782	−11.9	−2146	−9.2	635	3.1
39	Pontus Euxinus (perimeter including the straits)	Periplus Ponti Euxini	the sum of all the course legs of the route	23457,5	23858	24123	400	1.7	665	2.8	265	1.1
40	Africa from Tingis to Canobus[n]	Artemidorus	Agathem. 3.10; Plin. 5.40	29252	32105	31644	2853	9.8	2392	8.2	−461	−1.4
41	Africa from Tingis to Canobus	Isidore	Plin. 5.40	29576	32105	31644	2529	8.6	2068	7	−461	−1.4
42	Africa from Tingis to Canobus	Artemidorus (?)	Marcianus Hercl. 1.5	30280	32105	31644	1825	6	1364	4.5	−461	−1.4
43	Africa from Tingis to Canobus	Pliny	6.206	30384	32105	31644	1721	5.7	1260	4.1	−461	−1.4
44	Arabia from Elana to Charax	Iuba	Plin. 6.156	32000	37895		5895	18.4				
45	Arabia from Canobus to the Tanais	Pliny (Isidore ?)	6.156	38120	37895		−225	−0.6	−225	−0.6		
46	Asia from Canobus to the Tanais	Artemidorus, Isidore	Agathem. 3.10; Plin. 5.47	40111	41239	41057	1128	2.8	946	2.4	−182	−0.4
47	Europa without the Pontus (from Calpe to Byzantium)[s]	Artemidorus	Plin. 4.77,5.47, 6.206	56412	55924	58368	−488	−0.9	1956	3.5	2444	4.4
48	Europe from Calpe to the Tanais	Agathem., Isidore	Agathem. 3.10; Plin. 4.121	69709	71643	74770	1934	2.8	5061	7.3	3126	4.4
49	Mediterranean Sea without the Maeotis	Agrippa	F 36 Riese = Plin. 6.207	124072[t]	133792	136231	9720	7.8	12159	9.8	2439	1.8
50	Mediterranean Sea without the Maeotis	Artemidorus	Plin. 6.207	130120	133792	136231	3672	2.8	6111	4.7	2439	1.8
51	Mediterranean Sea including the Maeotis	Artemidorus	Agathem. 3.10	139072	144792	147471	5720	4.1	8399	6	2679	1.9

a F 31 Riese = Plin. 6.39: *oram omnem a Caso praedita rupibus accessum <ne~> gare per CCCCXXV p. auctor est Agrippa.* Ptolemy's coordinates for the mouth of the Casius are 82°30' long., 46° lat. and 84° long., 42°15' lat. for the mouth of the Cyrus.

b Contrary to the general tendency, there are good reasons to think that for Sicily, recession Ξ is reliable than Ω, because in Ξ Cape Pelorus (one of the three principal capes of Sicily) is omitted, and Segesta (the westernmost point) is shifted considerably further to the east, whereas in Ω the latter is quite reasonably placed on the same meridian with Ostia.

c Since Ptolemy locates the island of Gades too far from the coast, it would be more correct to measure this distance from the temple of Hera on the coast (see the Supplement).

d Codex Vaticanus Graecus 191 contains two clear errors. First, Paniardis is displaced far to the east (to 53°30' lat., instead of 53°30' lat., 67°30' in Ω) relative to the rest of the coast. Second, Parthenion and Myrmekion on the European coast of the Cimmerian Bosporus (modern Kerch Strait) are placed south of Achilles' sanctuary on its Asian side (at 48°15' lat. and 48°30' lat., respectively), in Ω, however, they are placed at the same latitude (at 48°30' lat., the latitude of the Borysthenes), which is more logical because Parthenion and Myrmekion are known to be situated opposite to Achilles' sanctuary in the narrowest part of the strait (Strabo 11.2.8 C494). Except for these two differences, Ξ matches with Ω almost exactly.

e *Natural History* 3.97: *A Lacinio promontorio secundus Europae sinus incipit, magno ambitu flexus et Acroceraunio Epiri finitis promuntorio, a quo abest LXXV;* 3.150: *universum autem sinum Italiae et Illyrii ambitu XVII.* The latter quotation relates immediately after the fragment of Agrippa about the coast of Illyria (F 13 Riese = F 16 Klotz; no. 1 in this table), which allows Riese and Klotz to ascribe it also to Agrippa (F 13 Riese=F 47 Klotz). Detlefsen (1886, 243 p., 247–248) has reasonably noted that this quotation is also connected with Pliny's passages about the so-called "second" and "third" gulfs of Europe (3.97; 4.1) which he ascribes to Varro, although they may well be attributed to Agrippa as well.

f In a number of cases, the inconsistencies in the Ξ data have been corrected: the positions of Cape Circei, Tarracina, Privernum, and Hidrentum are given according to Ω; of also Cuntz (1923, inserted map), Polaschek (1965, cols. 726–727, 730–731, inserted map). The coordinates of Tarentum (41°30' long., 40° lat.) are taken from the so-called *Canon of the noteworthy cities* (6.1). Stückelberger and Mittenhuber, 2009, p. 162); the outlines of the Gulf of Taranto are given according to the reconstruction proposed by Polaschek (1965, cols. 720–721, 730–731, map), even though it does not solve all contradictions in Ptolemy's data.

g "The provinces of Spain taken all together, measured from the two promontories of the Pyrenees along the sea line, are estimated to cover by the circumference of the whole coast 2924 miles, or by others 2600 miles", trans. by H. Rackham in the Loeb Classical Library (*omnes autem Hispaniae a duobus Pyrenaei promuntoriis per maria totius orae circuitu XXVIII ... XXIIII colligere existimantur, ab aliis XXVI*). It means from Cape Oiason to the Temple of Aphrodite in Ptolemy's *Geography*.

h Ξ gives coordinates only for the north-western coast of the Caspian Sea, therefore all the rest are taken from Ω. For arguments, see Detlefsen (1877, p. 24) and Klotz (1931, p. 441–442).

i Ξ omits the coordinates for the coast from Anthedon (64°50' long., 36°20' lat.), so they are taken from Ω.

j Pliny (4.77) lists other estimates of the circumference of the Pontus: 2150 m.p. = 17 200 stades according to Varro and "the old authorities generally" (*fere veteres*), 2500 m.p. = 20 000 stades in Cornelius Nepos, 2540 m.p. = 20 320 stades in Agrippa, and 2425 m.p. = 19 400 stades in Mucianus, Polybius (*History* 4.39.1) estimated it at 22 000 stades. Neither of these values agrees with Ptolemy's data.

k Klotz (1931, p. 441–442) emends Pliny's figure to XXVIII XXIII (2823 m.p. = 22 584 stades).

m However, the sum total stated in the *Periplus* (121[92]) is 23,587. For an explanation of some of the numerical errors underlying this discrepancy, see Diller (1952, p. 104).

n Remarkably, the sum of all distances given by Strabo for the coast of Libya amounts to a little more than 28 000 stades: 5000 from Cape Cotes to Cape Metagonium (17.3.6 C827), thence 6000 to Cape Tretum (17.3.9 C829), 2500 to Carthage (17.3.13 C832), more than 5000 to Cape Cephalae (in a straight line across the Lesser Syrtis) and thence 3930 along the Great Syrtis (17.3.18 C835), 1000 from Berenice to Apollonia (17.3.20 C837), 2200 to Catabathmus (17.3.22 C838), 900 to Paraetonium and 1300 to Alexandria (17.1.14 C798), 120 to Canobus (17.1.17 C801). With few exceptions, these distances disagree with Ptolemy's data, suggesting that Strabo and Ptolemy drew on different sources.

o Müller (1855, p. 520). It is tempting to emend Marcianus' figure ,ςε΄π (30 280) to ,ςφπ (30 380) which would be in accord with Pliny 6.208.

p *circuitus Arabiae a Charace Leviana colligere proditur XLVII LXV*, p. 520. Some secondary MSS read this figure as XLV LXV. p. Some secondary MSS read this figure as XLIV LXV = 36 520 stades or XLVIII LXV = 38 920 stades.

q Artemidorus' length of the coast of Europe can be derived from Anthedon (64°50' long., 36°20' lat.) to the border of Cilicia (69° long., 36°20' lat.), so they are taken from Ω.

s Ξ omits the coordinates for the coast of Europe from Anthedon (64°50' long., 36°20' lat.) to the border of Cilicia (69° long., 36°20' lat.), however, argues that this figure could not have come from Agrippa.

t The deviation of this value from that of Artemidorus is most probably due to the fact that Agrippa's estimate of the circumference of the Pontus was about 6000 stades shorter than according to Artemidorus (Plin. 4.77), Klotz (1931, p. 458–459), the coast of Africa (no. 40), and the coast of Asia from Canobus to Chalcedon according to Timosthenes (no. 34).
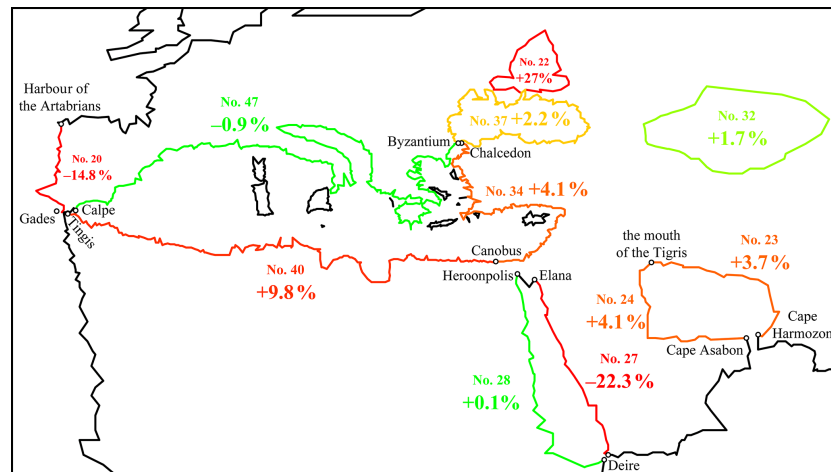
Figure 2. Differences between the coastline lengths as derived from Ptolemy's coordinates and as given by Artemidorus (including those that he has inherited from Timosthenes and Eratosthenes: nos. 20, 22–24, 27–28, 32, 40, 46–47 in Table 1). The percentages refer to the deviation of Ptolemy's values from those of Artemidorus. The outlines are drawn according to the Ξ recension supplemented with coordinates from Ω for the areas of Asia that are missing in Ξ.
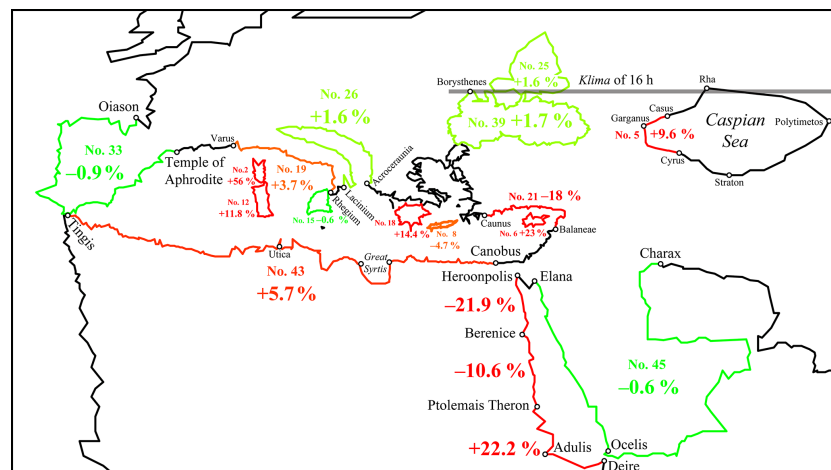

Figure 3. Differences between the coastline lengths as derived from Ptolemy's coordinates and as given by other *periploi*-based sources (nos. 2, 5–6, 8, 12, 16–19, 21, 25, 28, 33, 39, 43–44 in Table 1). The percentages refer to the deviation of Ptolemy's values from those of other sources. The outlines are drawn according to the Ξ recension supplemented with coordinates from Ω for the areas of Asia that are missing in Ξ.

omni varietate prodentium sumatur computatio), which implies that he knew even larger estimates.

Remarkably, Table 1 shows that the distances in Ξ are usually shorter than in Ω, but longer than in the other sources. This ratio can easily be explained by the so-called coastline paradox: the well-known fact that a coastline length varies depending on the level of detail of its geometry. The *periploi* described, of course, the coastal routes. However, these routes did not have to replicate all the minor, but numerous curves and bends of the coast. It is unsurprising, therefore, that the coastline in Ptolemy's *Geography*, when measured with all its curves, proves to be a little longer than the distances recorded in *periploi*, and Ω, as the second recen-

sion, has usually a more detailed and, therefore, even longer coastline than Ξ (Fig. 1).

5 The influence of Ptolemy's erroneous estimate of the circumference of the Earth

Distance data derived from *periploi* were bound to come into conflict with other Ptolemy sources, and particularly with his latitude data. In this case, contradictions must have been especially prominent because Ptolemy adopted an erroneous value for the Earth's circumference, namely 180 000 stades, which was about 17 % less than the actual value, if he used the stade of 185 m length (Shcheglov, 2016b). Owing to this
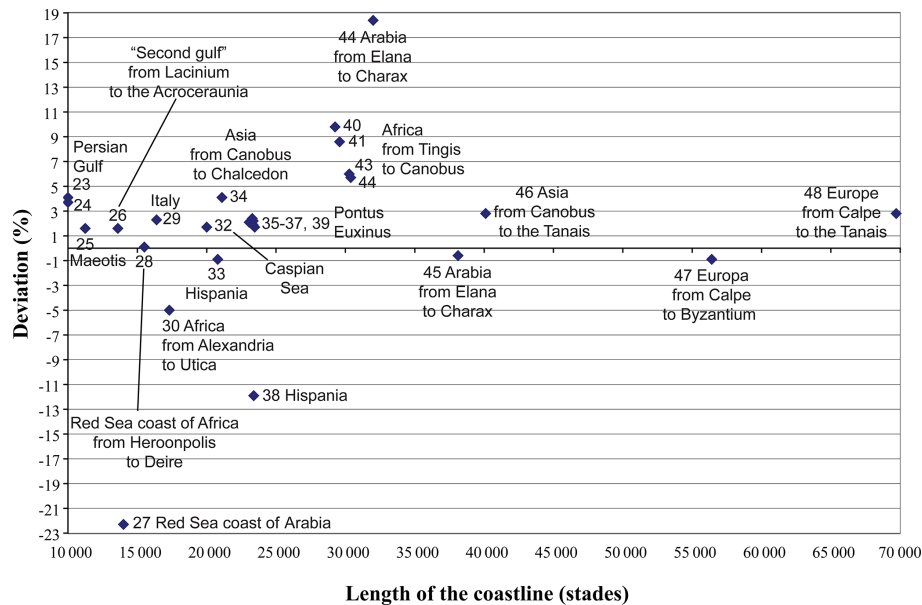
Figure 4. Deviation of Ptolemy's coastline length values from those given in the *periploi*-based sources for the distances over 10 000 stades (as expressed in the percentage terms). Ptolemy's data are mostly taken from Ξ and in a few cases from Ω (nos. 23–24, 27, 32, 44–45). Nos. 31, 49–51 are omitted since they are the sums of the other values.
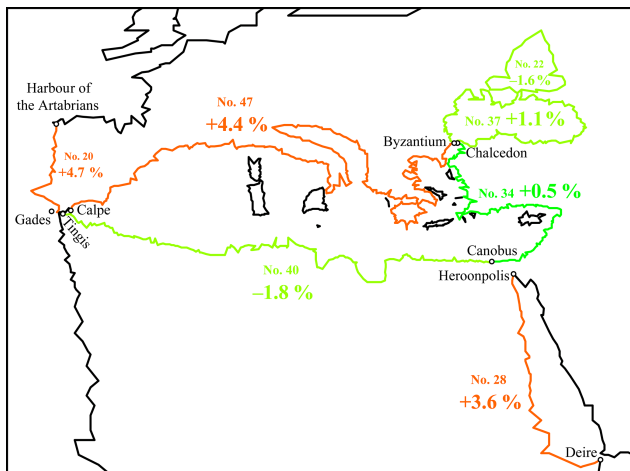

Figure 5. Differences between the coastline lengths in Ξ and in Ω as expressed in terms of percentage of the Ξ values (for the same coastal stretches as shown on Fig. 2).

error, all north–south distances which were tied to particular latitudes must also have been reduced by approximately the same amount. This effect may be illustrated by the example of the Red Sea coast of the Arabian Peninsula (no. 27 in Table 1). In Ptolemy's *Geography*, this coast is squeezed between the latitude of its northern extremity (Elana, 29°15′) and that of Ocelis (12°) near the Strait of Deire (Bab-el-Mandeb), the latter being based on astronomical observations (Ptol. *Geogr.* 1.7.4). It is unsurprising, therefore, that this coast exhibits a 22 % reduction in length.

However, I would like to draw attention to another possible and arguably more unusual side-effect of Ptolemy's underestimated circumference of the Earth. This effect is observed when a coastline in Ptolemy's *Geography* falls into two distinct parts: one oriented north–south, the other east–west. In a number of such cases the same pattern is seen: while the north–south part is shorter than the recorded *periploi* distance, which is understandable, the east–west part is proportionally longer, which requires explanation. It is reasonable to suppose that such a stretching of the east–west parts was intended to compensate for the shortening of the north–south part in order to maintain the total coastline length unchanged. If this explanation is valid, it can give us an important key to understanding Ptolemy's *modus operandi*.

The effect of proportional shortening/stretching of the neighbouring segments of the coast can be illustrated most clearly by three examples: Italy, the Caspian Sea, and the Red Sea. In the case of Italy, the southern part of the peninsula is squeezed between the latitudes of the Strait of Messina (at Rhegium) and Naples, whereas the part between Naples and Ostia is proportionally stretched from east to west (Fig. 7). The Caspian Sea is probably compressed from north to south together with the circumference of the Earth, but proportionally stretched from east to west (Figs. 2 and 3). In the case of the Red Sea, the largest part of its African coast is oriented from north to south and squeezed between the latitudes of its northern extremity and of Ptolemais Theron in the south, whereas its southern part, turning eastward to the Bab-el-Mandeb Strait, is proportionally stretched (Fig. 3). Below I consider these instances in detail.
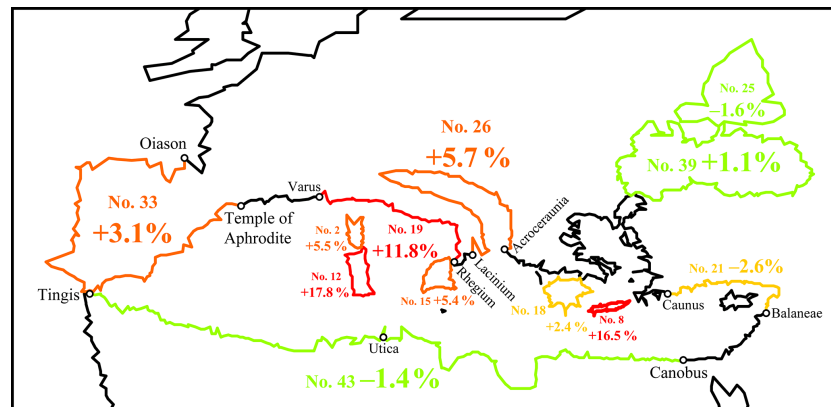
Figure 6. Differences between the coastline lengths in Ξ and in Ω as expressed in terms of percentage of the Ξ values (for the same coastal stretches as shown on Fig. 3).
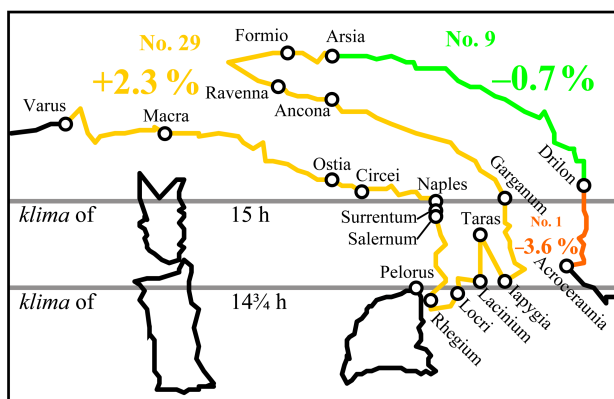

Figure 7. Pliny's points of the coast of Italy on Ptolemy's map (Ξ recension). The percentages refer to the deviation of Ptolemy's values for the coastline length from those given by Pliny (no. 29 in Table 1) and Agrippa (nos. 1 and 9).

Figure 8. Pliny's points of the coast of Italy on the modern map.

5.1 Italy

Only a few ancient sources provide detailed information on individual distances along the coast. Pliny's description of Italy is among them (*Natural History* 3.49, 51, 56, 62, 70, 73, 95, 97, 99, 100, 111, 115, 127). Table 2 shows how Pliny's values for the separate coastal stretches constituting the perimeter of Italy relate to Ptolemy's data (see also Figs. 3 and 7).

Ptolemy's Italy may be divided into three parts (Fig. 7): (1) the northern part lying west of Ostia and Ancona, (2) the middle part lying east of Ostia and Ancona and north of Naples, and (3) the part lying to the south of Naples. Table 2 shows that for northern Italy Ptolemy's distances accord tolerably with Pliny's figures.[17] Ptolemy's middle Italy

is distinctly stretched in the east–west direction relative to Pliny's distances, whereas southern Italy is almost equally compressed. For example, Ptolemy's coast from Circei to Surrentum is stretched by 78 m.p., whereas that from Salernum to Rhegium is shortened by 83 m.p. Similar compressions and stretchings of these coastal sections are exhibited by Ptolemy's map relative to the modern map (Table 2, Fig. 8).

The compression of southern Italy in the north–south direction can be easily explained, first of all, by Ptolemy's underestimated value for the Earth's circumference. Ptolemy's latitudes for southern Italy are rather accurate: for example, the interval between his latitudes of Surrentum and Rhegium

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[17]Ptolemy's coast from Ravenna to the mouth of the Formio matches Pliny's distances pretty well. The length of the coast from the Varus to Cape Circei is exaggerated by 8 % (or 42.5 m.p.) on

average as compared to Pliny's figures, which is quite a lot, but it can still be explained by the fact that Ptolemy's data almost always exhibit a similar overestimation of distances relative to the *periploi* data (Table 1).

Table 2. Comparison of the distances along the coast of Italy according to Pliny, Ptolemy, and measurements using Google Maps (Google Inc., 2017). All distances are expressed in terms of Roman miles; 1 m.p. = 1480 m.

Stretches of the coast from – to	Pliny (P)	Ptolemy		Difference between Pliny and Ξ		Google Maps*
		Ξ	Ω	m.p. (Ξ–P)	% (Ξ–P) / P × 100	
Varus–Macra	211	219.1	214.8	8.1	3.8 %	154–182
Macra–Tiber (Ostia)	284	315.1	350.4	31.1	11 %	215–240
Tiber (Ostia)–Cape Circei	50	53.2	53.2	3.2	6.5 %	58–61
Circei–Surrentum	78	155.6	224.2	77.6	99.5 %	87
Surrentum–Salernum	30	26.5	22.3	−3.5	−11.5 %	36–37
Salernum–Rhegium	237	153.7	167.8	−83.3	−35.1 %	200–236
Rhegium–Locri	66	73	71.7	7	10.7 %	100
Locri–Cape Lacinium	86 or 75	67.4	71.7	−18.6 or −7.6	−21.6 % or −10.1 %	83–95
Tarentum (Taras)–Iapygia	108	88	78.9	−20	−18.5 %	90–100
Iapygia–Garganum Mt	234	177.3	189.9	−56.7	−24.3 %	215–233
Garganum–Ancona	183	335.4	379.6	152.4	83.3 %	200–225
Ancona–Ravenna	105	88.3	88.6	−16.7	−15.9 %	90
Ravenna–Formio	189	183.9	187.9	−5	−2.7 %	155–190

* The accuracy of this comparison should not be overestimated, because the routes on Google Maps have been measured rather arbitrarily. The lower and upper values in the table refer to the shortest route across the open sea and the route along the coast, respectively.

is 2°25′, which is pretty close to the true value of 2°30′. However, for Ptolemy the corresponding distance amounts to 150 m.p., whereas the correct value would have been 188 m.p.

Ptolemy's southern Italy is distinctly squeezed between the latitudes of Naples (40°55′ in Ξ; the true latitude is 40°50′) in the north and that of the Strait of Messina (in particular Cape Pelorus, 38°35′ in Ω; the true latitude is 38°16′) in the south. These latitudes corresponded to the so-called *klimata* with the longest day of 15 and 14 3/4 h, respectively. *Klimata* (sing. *klima*) was a technical term for latitudes defined by the length of the longest day. These *klimata* were, in essence, the principal tool available to pre-Ptolemaic geographers for constructing a mathematically rigorous map of the world, at least, from Eratosthenes onwards.[18] A set of these *klimata* constituted the basis of the geographical system of Marinus of Tyre, Ptolemy's immediate predecessor and the principal source (Honigmann, 1930; Wurm, 1931). Therefore, Ptolemy's latitudes of Naples and Pelorus go back, most probably, to Marinus or even further.

It is striking that various divergences between Ptolemy's and Pliny's data on the separate coastal stretches (Table 2)[19] are in stark contrast with the close agreement between their values for the total length of the coast: Ptolemy's figures for the perimeter of Italy and for the southern side from the

mouth of the Varus to Rhegium deviate from Pliny's values by only 2.4 % (no. 29 in Table 1) and 3.7 % (no. 19). Hence, it is tempting to explain this by assuming that the magnitude and direction of the disagreements may have been selected intentionally so that stretchings and compressions would have cancelled each other out.

5.2 The Caspian Sea

The coincidence between Ptolemy's and Artemidorus' values for the perimeter of the Caspian Sea gives us a key to explaining its strange configuration in Ptolemy's *Geography*, namely why it is shown as being longer from east to west (ca. 8250 stades from Gangara to the mouth of the Polytimetos) than from north to south (ca. 4400 stades between the mouths of the Rha and the Straton; see Fig. 3).

First of all, it is tempting to connect this anomaly with the fact that the whole of Ptolemy's map exhibits a similar stretching of all its outlines from east to west relative to the modern map.[20] This stretching can largely be explained by Ptolemy's underestimated value for the Earth's circumference. This effect is due to the principal difference between the methods of measuring latitude and longitude (Shcheglov, 2016b; Graßhoff et al., 2016). Latitude can be determined by means of simple astronomical observations, which had been known to the Greeks since at least the 4th century BC, and starting at least with Hipparchus (2nd century BC) it was normally expressed in degrees. But it was not until the 18th century that an efficient and simple enough method

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[18]On the system of *klimata*, see Honigmann (1929), Neugebauer (1975, p. 43–45, 333–336, 725–733), Shcheglov (2004); on *klimata* as a basis of Ptolemy's map, see Wurm (1931, p. 20–21, 30), Isaksen (2013, p. 50, Fig. 3.4).

[19]Similar observations have been made by Mittenhuber (2012) for Ptolemy's Hispania.

[20]On the variation in this error for different parts of Ptolemy's map, see Shcheglov (2016b).

of determining longitude was devised. For an ancient geographer, the main method to find longitude was to convert distance measurements from customary linear units (Greek stades, Roman miles, etc.) to angular units (degrees), for which the estimate of the Earth's circumference was essential. This is why an error in this estimate inevitably affected all longitude values based on distance measurements, but had no effect on latitude values when they had been originally expressed in degrees. A too low value for the circumference of the Earth results in that all distances projected on its surface (i.e. converted to degrees) become proportionally overestimated in angular terms, which made the outlines on the map stretch mostly in the east–west direction (e.g. Russo, 2013; Shcheglov, 2016a, b). However, Ptolemy's underestimation of the size of the Earth could have been responsible for a stretching by only 17 %, which is evidently insufficient to explain the configuration of the Caspian Sea in his *Geography*.

Furthermore, Ptolemy's Caspian Sea is not only stretched from east to west but also compressed from north to south. The only ancient estimate of the north–south extension of the Caspian Sea comes from Strabo (2.1.17 C74): he defines the meridional distance between its southernmost end and the mouth entering the Ocean in the north as "about" (περὶ) 6000 stades. This value matches the actual straight-line length of the Caspian Sea pretty well (provided that 1 stade = 185 m). However, Ptolemy's latitude interval between the northern (the mouth of the Rha at $48°50'$) and the southern (the mouth of the Straton at $40°$) extremities of the Caspian Sea corresponds to only 4417 stades (Fig. 3; of course, there is no reason to suppose that any of Ptolemy's latitudes for the Caspian Sea could have been based on actual astronomical observations).

The situation is complicated by the fact that there may have existed a hypothetical early version of Ptolemy's *Geography* that was based on a different estimate for the Earth's circumference, namely 252 000 stades, which was ca. 16.5 % above the true value ($252 000 \times 185$ m = 46 620 km). This estimate was first proposed by Eratosthenes and can be regarded as almost generally accepted in antiquity. As has been shown by Schnabel (1930, p. 218–219), Ptolemy probably used this value for calculating geographic longitudes in the *Almagest*, one of his early works. Even more importantly, Sarre and Herzfeld (1911, p. 143–153), Wurm (1937, 1940), and Shcheglov (2004, 2017) have shown, independently of one another, that a large area on Ptolemy's map was built on the basis of Eratosthenes' set of distances which were converted to spherical coordinates according to Eratosthenes' rate of $1° = 700$ stades. This area covers the whole of the Middle East, at least, between the Euphrates and the Indus including the Caspian Sea. More importantly, Ptolemy's breadth of the Caspian Sea, when expressed in Eratosthenes' 700-stade degrees, corresponds to 6183 stades which closely agrees with Strabo's distance, as was noted by Wurm (1937, p. 7, 13, 1940, p. 8, 11).

It is also remarkable that, on Ptolemy's map, the northern extremities of the Caspian Sea, as well as the northern coast of the Pontus Euxinus, are clearly tied to the latitude of the mouth of the Borysthenes ($48 1/2°$), one of the seven *klimata* or the key latitudes determining the structure of his map and going back, most probably, as far as Eratosthenes (Fig. 3).[21] These correspondences make it possible to suppose that Ptolemy's meridional extension of the Caspian Sea had been originally defined as slightly more than 6000 stades and that this distance was expressed in degrees of latitude already in the early version of the map, which was based on Eratosthenes' value for the Earth's circumference.

Interestingly, 6183 stades is quite close to the quotient of 20 000 stades (the perimeter of the Caspian Sea according to Artemidorus; no. 32 in Table 1) divided by π. If this coincidence is not accidental, and if the circumference of 20 000 stades and the meridional extension of slightly more than 6000 stades were the two parameters that determined the outlines of the Caspian Sea from the very beginning, then it mathematically implies that in the early "Eratosthenic" version of Ptolemy's map the sea must have had a round shape. We may suppose that, when Ptolemy had accepted a smaller value for the Earth's circumference instead of Eratosthenes' 252 000 stades, the meridional extension of the Caspian Sea, being expressed in degrees, remained unchanged, whereas its circumference was recalculated to the 500-stade degrees, which made the sea stretch from east to west and take the shape it has on Ptolemy's map today.

There is a difficulty, however, in that other ancient sources never compare the Caspian Sea with a circle.[22] On the contrary, they often use quite different comparisons: a sickle or the horns of the crescent, or a mushroom cap.[23] But this choice of metaphors might be due to the fact that, unlike Ptolemy, other sources considered the Caspian Sea as a gulf of the Ocean. Anyway, a circle seems to be the simplest and the most obvious figure that comes to mind to describe the shape of an unexplored sea or gulf about which almost nothing was known.[24]

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[21]On Eratosthenes as a source of Ptolemy's seven *klimata*, see Honigmann (1929) and Shcheglov (2004).

[22]In contrast to, for example, the Persian Gulf which was described as "round" and "similar to a human head", see Agathem. 3.12: κυκλοτερής; Mela, *Chorogr.*, 3.71: *reddit formam capitis humani*; Plin. 6.108: *humani capitis effigie*.

[23]Agathem. 3.13: μηνοειδής (crescent-shaped); Curt. Ruf. 6.4.16: *...lunae maxime similem, quum eminent cornua, quum eminent cornua, nondum totum orbem sidere implente*; Plin. 6.38: *...lunatis obliquatur cornibus...sicilis, ut auctor est M. Varro, similitudine*; Iordan. *Getica* 5.30: *ab Oceano euroboro in modum fungi, primum tenui, posthaec latissima et rotunda forma exoritur*; the same crescent-like shape of the Caspian Sea is seen on the so-called Tabula Peutingeriana, see http://www.cambridge.org/us/talbert/mapb/TP2000seg11.jpg or http://peutinger.atlantides.org/map-a.

[24]Remarkably, there are two other seas that have regular rounded shapes in Ptolemy's *Geography*: the Bay of Ganges (modern Bay

Table 3. Length of the Red Sea coast according to Artemidorus, Eratosthenes, Ptolemy, and measurements using Google Maps (Google Inc., 2017). All distances are expressed in stades; 1 stade = 185 m.

Stretches of the coast	Distance (stades)				Difference between Ξ and Artemidorus		Difference between Ξ and Google Maps	
	Eratosthenes and Artemidorus (D)	Ptolemy		Google Maps (GM)	stades	%	stades	%
		Ξ	Ω		(Ξ–D)	(Ξ–D)/ D × 100	(Ξ–GM)	(Ξ–GM)/ GM × 100
Heroonpolis–Ptolemais	9000	7575	7864	8108–8189	−1,425	−15.8	−533–614	−6.5–7.6
Heroonpolis–Berenice	4180	3266	3366	4324	−914	−21.9	−1058	−24.5
Berenice–Ptolemais	4820	4310	4497	3784–3865	−510	−10.6	445–526	11.5–13.9
Ptolemais–Deire	6500	7944	8211	4700–4843	1444	22.2	2766–2909	57.1–61.9

5.3 The Red Sea

The general configuration of the Red Sea (or, in ancient terms, the Arabian Gulf) on Ptolemy's map is determined by the latitudes of four points: Berenice, Ptolemais Theron, Adulis, and Ocelis (Fig. 3).[25] The length of the African coast from Heroonpolis (the northernmost point) to Deire (situated opposite to Ocelis) in Ptolemy's work is exactly the same as in Artemidorus (in Ξ it is only 0.1 % or 19 stades shorter, in Ω it is 3.7 % or 574 stades longer; see Table 1). However, the lengths of the separate coastal stretches recorded by Eratosthenes and Artemidorus relate to Ptolemy's data as follows.

Table 3 shows that the north–south stretch from Heroonpolis to Ptolemais is ca. 1425 stades shorter in Ptolemy than according to Artemidorus, while the next one turning to the east from Ptolemais to Deire is ca. 1444 stades longer.[26] Similar deformations of these stretches are exhibited by Ptolemy's

map relative to the modern map (Table 3).[27] As in the case of Italy, the compression of the north–south coastal segment can be accounted for by Ptolemy's erroneous estimate of the circumference of the Earth, but the stretching of the segment Ptolemais–Deire by exactly the same amount seems suspicious and requires some additional explanation.

6 Conclusions and further perspectives

The main conclusion of our study is intuitively expectable and in this sense unsurprising, namely that Ptolemy's *Geography* was most likely based on some ancient *periploi* similar to those known from other sources. What seems unexpected and much more important is that our study has revealed a close numerical agreement between Ptolemy's *Geography* and the other *periploi*-based sources in the data on the length of relatively long coastlines (over 10 000 stades): in most cases the differences between Ptolemy's data and the other sources fall between +4 and −1 %. This observation can provide a new insight into the genesis of Ptolemy's *Geography* and the history of ancient geography in general.

A major challenge we face now is how to reconcile this striking agreement between Ptolemy's data and the other sources with the equally striking contradictions between them in the short coastal stretches of which the long ones consist (as was demonstrated by the example of Italy). In other words, with increase in the coastline lengths being compared, different discrepancies between Ptolemy and the other sources compensate one another until an almost complete agreement is reached. But what factors determined this transformation? Could it be intentional, or just accidental, and to what extent? This question compels us to revisit our understanding of Ptolemy's methods and the relevancy of our methods of examining his work. For the time being, we

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of Bengal) and the Great Bay (the Tongking Gulf); see e.g. Stückelberger and Graßhoff (2006, p. 896–897, 902–903).

[25] In Ptolemy's *Geography*, Berenice is situated at the latitude of Syene (23°50′), Ptolemais at the latitude of Meroe (16°30′ in Ξ and in *Canon of the noteworthy cities* 14.2 or 16°25′ in Ω), which correspond to the *klimata* with the longest day of 13 1/2 and 13 h, respectively. Such arrangement of these points goes back as far as Eratosthenes (F IIB37–38, IIIA18 Berger = F 41–42, 59 Roller) or, at least, Hipparchus (F V4–V5 Berger = F 46–47 Dicks = Strab. 2.5.36 C133). Adulis (*Almagest* 2.6.4 Heiberg p. 105–106) and Ocelis (*Geogr.* 8.22.7) were presumably connected with the *klima* of the 12 3/4 h day (12°30′) in the early version of Ptolemy's map (see Wurm, 1931, 23–24), but in the final version they have been shifted to the latitudes of 11°40′ (Ξ and *Canon of the noteworthy cities* 14.2) or 11°20′ (Ω) and 12°, respectively. Modern latitudes of these points (according to the database of http://pleiades.stoa.org) are as follows: Berenice: 23°55′, Ptolemais: 18°13′, Adulis: 15°13′, Ocelis: 12°43′, Deire: 12°26′.

[26] Artemidorus gave 9000 stades for the distance from Heroonpolis to Ptolemais, and thence 6500 to Deire (Agathem. 3.14). However, according to Pliny (6.164), Artemidorus' values for the first distance was 9500. Eratosthenes gave also 9000 stades from Heroonpolis to Ptolemais, thence 4500 to Deire (F IIIB48 Berger = F 95 Roller = Strab. 16.4.4 C768), 4820 from Berenice to Ptolemais

(F IIB37–38 Berger = F 41–42 Roller = Plin. 2.183; 6.171), and, accordingly, 4180 from Heroonpolis to Berenice.

[27] However, a part of this deformation is due to the fact that Ptolemy and his predecessors placed Ptolemais 2° too far to the north of Marsa Aqiq, its modern counterpart.

may only suggest a preliminary explanation. Most plausibly, Ptolemy's map was constructed not "bottom-up" – when its outlines would have been drawn successively point by point like beads are strung on a thread, but "top-down" – when the cartographer first drew the general outlines determined by long distances, and only after that he inserted minor details, fitted into the general picture. This assumption can be corroborated in part by the three cases (Italy, the Caspian Sea, and the Red Sea) where Ptolemy's *Geography* exhibits a proportional shortening of the north–south sections of coastline and stretching of the neighbouring east–west sections relative to the distances recorded in other sources. Whereas the shortening of the north–south sections was definitely caused by Ptolemy's underestimate of the circumference of the Earth, the stretching of the east–west sections is hard to explain, unless by assuming that the cartographer deliberately compensated the shortening. Of course, three cases are not enough for definitive conclusions, and the issue awaits further investigation.

Appendix A: Pliny's "third gulf" of Europe

In addition to the materials presented in Table 1, one case needs to be considered separately. Pliny (4.1), without indicating his source, estimates the perimeter of the so-called "third gulf" of Europe from the Acroceraunia Promontory (modern Karaburun) to the Hellespont (Dardanelles) at 1925 m.p. = 15 400 stades "without minor gulfs" (*praeter minores sinus*). Since it is not known which gulfs he regarded as "minor", it is hazardous to rely on this evidence. Nevertheless, we can try, as an experiment, to measure the coast from the Acroceraunia to Sestos (as the major point on the Hellespont) in Ptolemy's *Geography* without the most prominent of the "minor gulfs" mentioned by Pliny (Fig. A1).[28] Without these gulfs, the coastline length measures 15 778 stades in Ξ or 15 714 stades in Ω, which is quite close to Pliny's figure. When measured including all the gulfs, it would amount to 21 552 stades in Ξ or 21 589 in Ω.
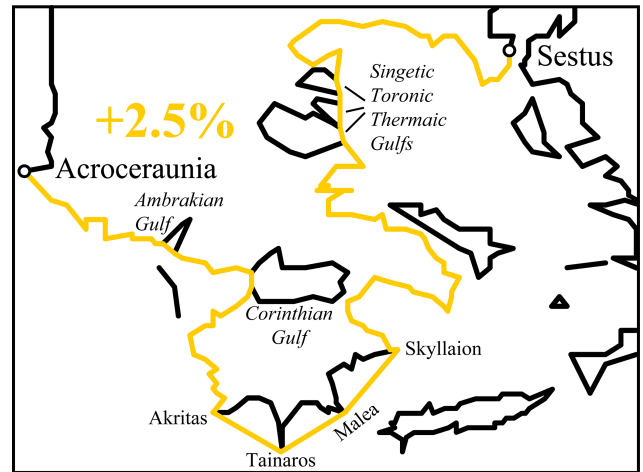

Figure A1. The outlines of Pliny's "third gulf" of Europe on Ptolemy's map (Ξ recension) without the "minor gulfs" (shown with yellow).

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[28]The Ambracian Gulf (between Nicopolis and Leucas), the Corinthian (between capes Antirrhion and Drepanon), the Messenian (between capes Akritas and Tainaros), the Laconian (between capes Tainaros and Malea), the Argolic (between capes Malea and Skyllaion), the Thermaic, Toronic, and Singitic gulfs (of course, this choice is rather arbitrary).

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