Hist. Geo Space Sci., 7, 27–52, 2016
www.hist-geo-space-sci.net/7/27/2016/
doi:10.5194/hgss-7-27-2016

Open Access History of Geo- and Space Sciences

The western coast of Africa in Ptolemy's *Geography* and the location of his prime meridian

Christian Marx

Gropiusstraße 6, 13357 Berlin, Germany

Correspondence to: Christian Marx (ch.marx@gmx.net)

Received: 5 October 2015 – Revised: 10 January 2016 – Accepted: 23 February 2016 – Published: 11 March 2016

Abstract. A controversial question concerning ancient geography is the location of the prime meridian which underlies the position data in Ptolemy's *Geography* and runs through the Fortunate Islands. An answer to this question is derived by means of a localisation of the places given by Ptolemy at the African western coast, i.e. in *Mauritania Tingitana* and *Libya Interior*, whose modern identifications are often uncertain or unknown. The origination of Ptolemy's positions from the distance data of seafarings is considered. A comparison of his data with distances reported by Pliny yields a satisfactory agreement. The localisation of Ptolemy's places is based on distances derived from Ptolemy's coordinates and partly on further information by ancient authors. Through it previous identifications are confirmed and new identifications are found. It shows that the Fortunate Islands correspond to several eastern islands of the Canary Islands. Explanations are given for the errors of Ptolemy's position data. A likely error by Ptolemy barely considered is his repetition of a part of *Mauritania Tingitana* in his description of *Libya Interior*. The existence of this repetition is confirmed by an adjustment of a transformation between the positions of the duplicated places and a statistical test of the arranged model. A comparison of longitudinal distances in different ancient sources reveals that the position of Ptolemy's prime meridian is based on distances given by Marinos and Eratosthenes.

1 Introduction

The search for the origin of the concept of geographic coordinates (longitude and latitude) for the specification of a position on the earth's surface leads to Eratosthenes (ca. 276–194 BC), whose works, however, are lost. His geographical knowledge is mainly handed down by Strabo's (ca. 63 BC–AD 23) *Geography* (G; see e.g. Jones, 1917–1932; Aujac et al., 1969–2015), where longitudinal and latitudinal distances originating from Eratosthenes are reported (see Roller, 2010; Marx, 2015). Hipparchus (ca. 190–120 BC) applied spherical coordinates in astronomy and it seems likely that he used them also in his geographical works (cf. Dicks, 1960, 148–149). The first known discussion of the preparation of a graticule by means of meridians and parallels is given by Strabo (Berggren and Jones, 2000, p. 32). Probably the first comprehensive use of geographic coordinates is to be found in Ptolemy's (Klaudios Ptolemaios, ca. AD 100–170) *Geography* (*Geographike Hyphegesis*, GH). In a catalogue of locations in books 2–7 of the *Geography*, the positions of several thousand places are given by means of longitude and latitude stated in degrees.

The counting of the latitude is physically connected with the Equator, whereas the specification of the longitude necessitates the definition of a reference (prime) meridian. Eratosthenes used an assumed meridian through *Alexandria*, *Rhodos*, *Byzantion* and other places wrongly assigned to it (G 1.4.1, 2.5.7). A precursor of this meridian is to be found in the map created by Dicaearchus (ca. 326–296 BC), which passed approximately through *Rhodos* (Aujac et al., 1987). Eratosthenes' prime meridian was adopted by Hipparchus (cf. G 1.4.1). Ptolemy also uses a prime meridian running through *Alexandria* in his *Mathematike Syntaxis* (MS; see Toomer (1984), e.g. MS 4.6) and in GH Book 8, but owing to his advanced geographical knowledge he does not locate the places at this meridian, which were wrongly assigned to it by Eratosthenes. For the catalogue of locations, Ptolemy applies a different prime meridian west of the African Atlantic coast which runs through the *Insulae Fortunatae* (Fortunate

Islands). These islands are usually equated with the Canary Islands (cf. Keyser, 1993; Stückelberger and Graßhoff, 2006, p. 14, fn. 38); yet another frequent identification is the archipelago of Cape Verde (e.g. Spaul, 1958, p. 8.10; Lacroix, 1998, p. 202).

An objective of the present contribution is to give an answer to the question of the location of Ptolemy's prime meridian. To this end, the Fortunate Islands are localised based on the places given by Ptolemy at the African western coast, which is described in connection with Africa in GH Book 4. This concerns the western coast of *Mauritania Tingitana* in GH 4.1 and of *Libya Interior* in GH 4.6. The modern counterparts of the majority of Ptolemy's places at the African western coast are not known for certain. For example, the Barrington Atlas (Talbert, 2000) only covers the northern part of Morocco and localises a few of Ptolemy's coastal places only. In the edition of the *Geography* by Stückelberger and Graßhoff (2006, referred to as S & G), modern identifications are not specified in many cases. Differing localisations are to be found in several treatises, e.g. Gosselin (1798–1813, 120–64), Mannert (1825, 467–632), Forbiger (1844, 862–92), Spaul (1958) and Lacroix (1998).

In the recent past, the places of GH books 2 and 3 have been identified, supported by the geodetic–statistical analysis method by Marx (2012); see e.g. Marx and Kleineberg (2012). This approach, however, is not suitable for the coastal regions of *Mauritania Tingitana* and *Libya Interior*, since the longitudinal differences of places are erroneous and the density of the locations is too low for the most part. Spaul (1958) locates Ptolemy's places of *Mauritania Tingitana* by means of distances derived from Ptolemy's coordinates. This approach is improved and applied in the present contribution. It is based on the coordinate values of the Ω recension (Ω) and the Ξ recension, represented by the manuscript *Codex Vaticanus Graecus* 191 (X), according to S & G. In addition to maps, the localisation was supported by internet-based mapping applications such as Google Earth (Google Inc., 2015). Gosselin (1798–1813) assumes that places of the African western coast are repeatedly given by Ptolemy. This hypothesis is supported by e.g. Mauny (1978) but is not taken into account by the works mentioned above, and is reconsidered in the following. Pliny's *Natural History* (NH; see Bostock and Riley, 1855; Winkler and König, 1993; Brodersen, 1996) and the *Periplus* of Hanno (e.g. Lendering, 1998–2014) are consulted as further ancient sources of information.

Initially, Ptolemy's coordinates and their origin are considered in Sect. 2. In Sect. 3, the applied calculational methods for the localisation of places are described. In Sect. 3.2, possible errors of the Ptolemaic distances are pointed out and the data are compared with distances given by Pliny. In Sect. 4, the places of the African western coast are localised. The identity of places in GH 4.1 and GH 4.6 is investigated by means of a statistical test in Sect. 4.4. In Sect. 5, the errors of Ptolemy's data are analysed. In Sect. 6, the Fortunate Islands are localised. In Sect. 7, the origin of the position of Ptolemy's prime meridian is investigated. In addition, the findings reveal Ptolemy's determination of the longitude of *Byzantion*, which is given in Appendix D.

Appendix E lists the abbreviations used. Place names are based on S & G and Bostock and Riley (1855). Excerpts from the ancient sources are taken from the translations indicated.

2 Ptolemy's data

In the following, Ptolemy's locations dealt with by the present investigation are considered (Sect. 2.1) as well as the origination of their position data (Sect. 2.2). The places of GH 4.1 *Mauritania Tingitana* and GH 4.6 *Libya Interior* with given coordinates have been numbered consecutively according to their appearance in the manuscripts (see Tables 1–3). If necessary, the place numbers of GH 4.1 and 4.6 are distinguished by the prefixes "M" and "L", respectively.

2.1 Places

Figures 1 and 2 show the places of GH 4.1 *Mauritania Tingitana* and of the western part of GH 4.6 *Libya Interior* based on the Ptolemaic coordinates of Ω. The Ptolemaic longitudes Λ and latitudes Φ of the considered places are given in Tables 1–3. Only the coordinate variant which showed to agree better with the actual position is specified; further values are given in Sect. 4.

Mauritania Tingitana was a Roman province which corresponds to the northern part of Morocco. Pliny mentions *Volubilis* (at Moulay Idriss) in the inland and *Sala* (Chellah) at the western coast as the southernmost places in his description of *Mauritania Tingitana* (NH 5.1, division according to Bostock and Riley, 1855). These places are also given by Ptolemy (nos. 57 and 7). He describes the western side of *Mauritania Tingitana* in GH 4.1.1–4 (nos. 1–24). (59) *Tokolosida* (Bled Takourart, S & G) in the inland near the coast was the southernmost place of the Roman territory (Forbiger, 1844, p. 878, fn. 14). Ptolemy's description of the coast, however, contains several places which are situated further south. The southernmost (24) Bigger *Atlas* Mountain at $\Phi = 26°30'$ is situated about 7° more southerly than *Volubilis* and *Sala*. Ptolemy gives the south-eastern border point of *Mauritania Tingitana* at $\Phi = 26°$ (GH 4.1.8). Consequently, Ptolemy's *Mauritania Tingitana* ranges much further south than the Roman province (also Mannert, 1825, p. 473).

Ptolemy describes the coast of *Libya Interior* in GH 4.6.5–7 (nos. 3–25). He states (4.6.4) that in the north its western side reaches up to the border point of *Mauritania Tingitana* (at $\Phi = 26°$; see above), and this is in agreement with his latitudes in *Libya Interior*, which are lower than those of *Mauritania Tingitana* (see Figs. 1 and 2). The southernmost location is the (2) Hesperian Gulf at $\Phi = 4°$.

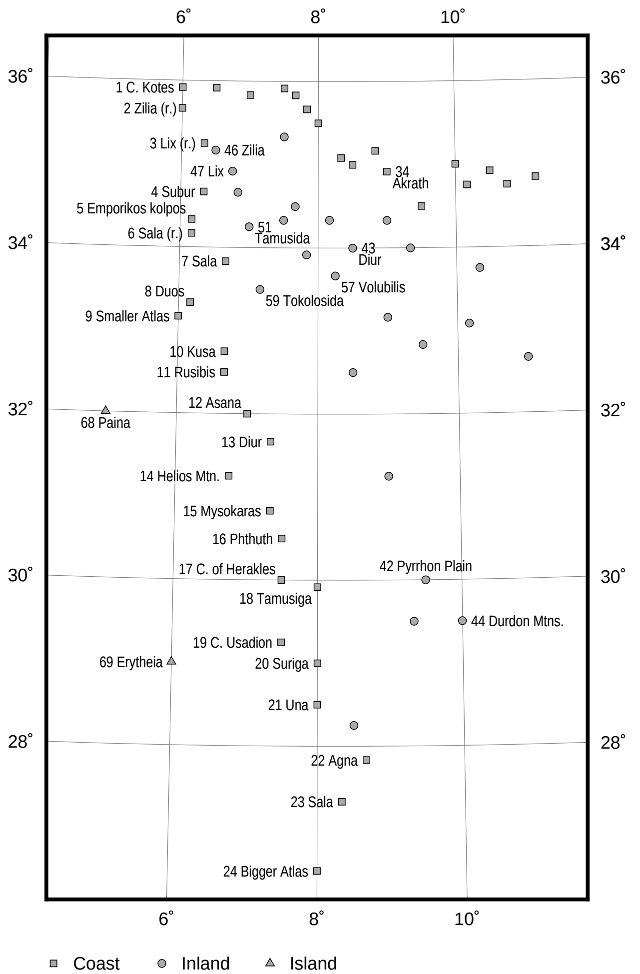

Figure 1. Ptolemy's positions in GH 4.1 *Mauritania Tingitana* (Ω).

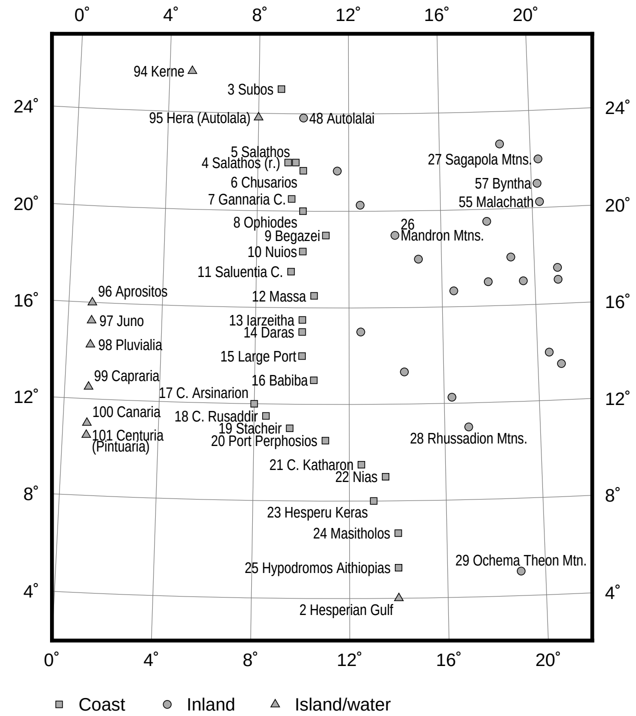

Figure 2. Ptolemy's positions in the western part of GH 4.6 *Libya Interior* (Ω).

2.2 Origination of coordinates

(M1) Cape *Kotes*/Cap Spartel and the (M3) *Lix*/Uad Lucus, the northernmost places at the western coast, have almost correct latitudes. They may be based on direct measurements at the place or in its vicinity (gnomon measurement). One kind of Ptolemy's data sources may have been itineraries, but this is only likely for the region of the Roman province *Mauritania Tingitana*, that is, for the places up to (M7) *Sala*. The majority of Ptolemy's positions at the coast is surely based on the information of the descriptions of sea routes along the coast. Known seafarings are the circumnavigation of Africa by Phoenicians under orders from Necho II (ca. 600 BC) and the voyages of Hanno (5th century BC), Polybios and Eudoxos of Kyzikos (2nd century BC) along the African western coast (e.g. Mauny, 1978; Keyser, 1993).

Travel reports contained journey times (see e.g. Hanno's *Periplus*), which were converted into distances by means of assumptions about the speed. Either such estimated distances were available to Ptolemy or he himself converted journey times. Distance data may have been corrected for assumed errors. Ptolemy used to shorten distances in order to take into account bends of routes and anomalies of journeys, and in his examples a reduction of $\frac{1}{3}$ is applied for each of these corrections (GH 1.2.4, 1.13). Differences of coordinates were derived based on Ptolemy's setting

$$1^\circ \mathrel{\hat{=}} 500\,\mathrm{st}. \tag{1}$$

(GH 1.7.1; st = stade) and further information or assumptions about the direction of the route. Berggren and Jones (2000, 16–17) give an overview of Ptolemy's procedures of the determination of coordinates.

According to GH 1.9.4 and 1.18.10, distances were specified as the number of day-and-night seafarings (DN) and day distances (D) (cf. S & G, p. 105, fn. 117), and it applies

$$1\,\mathrm{DN} = 2\,\mathrm{D}. \tag{2}$$

Concerning the speed, Ptolemy says that one must trust the feasible daily shipping performance (GH 1.9.6). Forbiger (1842, p. 550) and Kroll (1921) give overviews of the specifications of ancient authors about the speed of seafarings. Ptolemy, referring to other sources, mentions the speed (GH 1.9.4)

$$v_{\varnothing} = 1000\,\mathrm{st\,day}^{-1}, \text{ i.e.} \tag{3}$$

$$1\,\mathrm{DN} = 1000\,\mathrm{st}, \tag{4}$$

which also results from G 10.4.5 and which is a serviceable average value (similarly Forbiger, 1842, p. 551). The length

Table 1. Places of Ptolemy's *Geography* at the African western coast as well as further inland places and their identifications, part 1. S.: source of Λ and Φ.

No.	Ancient name	Λ (°, ′)	Φ (°, ′)	S. Λ, Φ	Identification	λ (°, ′)	ϕ (°, ′)
GH 4.1 *Mauritania Tingitana* – western coast							
1	Cape *Kotes*	6, 00	35, 55	Ω, Ω	Cap Spartel	−5, 56	35, 47
2	River *Zilia*	6, 00	35, 20	Ω, *X*	Oued el Gharifa	−6, 00	35, 32
3	River *Lix*	6, 20	35, 15	Ω, Ω	Uad Lucus	−6, 09	35, 12
4	River *Subur*	6, 20	34, 40	Ω, Ω	Oued Sebou	−6, 41	34, 16
5	*Emporikos kolpos*	6, 30	34, 20	*X*, Ω	Oued Sebou – Oued Bou Regreg	−6, 45	34, 09
6	River *Sala*	6, 30	34, 10	*X*, Ω	Oued Bou Regreg	−6, 51	34, 02
7	*Sala*	6, 40	33, 50	Ω, Ω	Chellah (Rabat)	−6, 50	34, 01
8	River *Duos*	6, 10	33, 40	Ω, *X*	Oued Cherrat	−7, 07	33, 50
9	Smaller *Atlas*	6, 00	33, 10	Ω, Ω	Moroccan Meseta west of the Middle Atlas	−6, 49	33, 36
10	River *Kusa*	6, 40	32, 45	Ω, Ω	Oued Mellah	−7, 25	33, 42
11	Port *Rusibis*	6, 40	32, 30	Ω, Ω	Casablanca	−7, 38	33, 36
12	River *Asana*	7, 00	32, 00	Ω, Ω	Oued Oum er-Rbia	−8, 20	33, 19
13	River *Diur*	7, 20	31, 40	Ω, Ω	Sidi Moussa lagoon	−8, 45	32, 59
14	*Helios* Mtn.	6, 45	31, 15	Ω, Ω	Cap Cantin	−9, 17	32, 33
15	Port *Mysokaras*	7, 20	30, 50	Ω, Ω	Safi	−9, 15	32, 18
16	River *Phthuth*	7, 30	30, 30	Ω, Ω	Oued Tensift	−9, 21	32, 02
17	Cape of Herakles	7, 30	30, 00	Ω, Ω	Cap Hadid	−9, 41	31, 42
18	*Tamusiga*	8, 00	29, 55	Ω, Ω	Essaouira (Mogador)	−9, 46	31, 31
19	Cape *Usadion*	7, 30	29, 15	Ω, Ω	Cap Sim	−9, 51	31, 23
20	*Suriga*	8, 00	29, 00	Ω, Ω	–	–	–
21	River *Una*	8, 00	28, 30	Ω, Ω	Oued Iguezoullene	−9, 49	31, 05
22	River *Agna*	8, 40	27, 50	Ω, Ω	Oued Tamri	−9, 51	30, 43
23	River *Sala*	8, 20	27, 20	Ω, Ω	Oued Sous	−9, 36	30, 22
24	Bigger *Atlas*	8, 00	26, 30	Ω, Ω	Western foothills Anti-Atlas	−9, 50	29, 48
GH 4.1 *Mauritania Tingitana* – further places							
42	*Pyrrhon* Plain	9, 30	30, 00	Ω, Ω	Haouz plain	−8, 00	31, 30
43	*Diur*, middle	8, 30	34, 00	Ω, Ω	Rif, middle	−4, 45	34, 50
44	*Durdon* Mtns., western part	10, 00	29, 30	Ω, Ω	High Atlas around Toubkal	−7, 55	31, 04
46	*Zilia*	6, 10	35, 10	*X*, Ω	Dchar Jedid	−5, 55	35, 31
47	*Lix*	6, 45	34, 55	Ω, Ω	(At) Larache	−6, 07	35, 12
51	*Tamusida*	7, 00	34, 15	Ω, Ω	Sidi Ali ben Ahmed	−6, 30	34, 20
57	*Volubilis*	8, 15	33, 40	Ω, Ω	At Moulay Idriss	−5, 32	34, 03

of the stade underlying ancient data is often unknown, which also applies to Ptolemy's coordinates. Therefore two measures of the stade are applied in the following, by which the major range of the measures is covered. These are the $\frac{1}{8}$ Rmi or Italian stade (Ist) according to G 7.7.4 and the Egyptian stade (Est) according to Hultsch (1882, p. 61):

$$1\,\mathrm{Ist} = 185.2\,\mathrm{m} \quad (5)$$

$$1\,\mathrm{Est} = 157.5\,\mathrm{m} \quad (6)$$

(Rmi = Roman mile, 1 Rmi = 1.4815 km). The former was probably used by Polybios (Pothecary, 1995), who may have been a source for Ptolemy's African coast, and both stades have been ascribed to Eratosthenes (see e.g. Pothecary, 1995). From Eq. (4) it follows that 1 DN = 185/158 km, 1 D = 93/79 km, $\frac{1}{2}$ D = 46/39 km (Ist/Est).

3 Method of localisation

The unknown position of a place at the coast is determined uniquely by the distance from a known place and an indication of the direction along the coast. This information can be derived from the Ptolemaic coordinates so that they can be used for the localisation of the Ptolemaic places. In the following, the reconstruction of the ancient distances (Sect. 3.1) and the accuracy of reconstructed distances (Sect. 3.2) are considered.

3.1 Determination of distances

Seafarings along the African western coast were influenced by ocean currents and winds. The present current north of 10° N is the Canary Current flowing in the southwesterly

Table 2. Places of Ptolemy's *Geography* at the African western coast as well as further inland places and their identifications, part 2. S.: source of Λ and Φ.

No.	Ancient name	Λ (°, ′)	Φ (°, ′)	S. Λ, Φ	Identification	λ (°, ′)	ϕ (°, ′)
GH 4.1 *Mauritania Tingitana* – further places (continuation)							
59	*Tokolosida*	7, 10	33, 30	Ω, Ω	Bled Takourart	−5, 36	34, 02
61	*Molochath*	10, 10	33, 05	Ω, Ω	–	–	–
62	*Benta*	9, 30	32, 55	Ω, *X*	–	–	–
68	Island of *Paina*	5, 00	32, 00	Ω, Ω	Island of Sidi Abderrahmane	−7, 42	33, 35
69	Island of *Erythia*	6, 00	29, 00	Ω, Ω	Mogador	−9, 47	31, 30
–	Mtn. *Phokra*	–	–	–	Southern part of the Moroccan Meseta	−8, 00	32, 30
GH 4.6 *Libya Interior* – western coast							
2	Hesperian Gulf	14, 00	4, 00	Ω, Ω	Gulf of Guinea	4, 00	1, 30
3	River *Subos*	9, 00	25, 00	Ω, Ω	Oued Sebou	−6, 41	34, 16
4	River *Salathos*	9, 20	22, 00	Ω, Ω	Oued Bou Regreg	−6.51	34, 02
5	*Salathos*	9, 40	22, 00	Ω, Ω	Chellah (Rabat)	−6, 50	34, 01
6	River *Chusarios*	10, 00	21, 40	Ω, Ω	Oued Mellah	−7, 25	33, 42
7	*Gannaria* Capes	9, 30	20, 30	Ω, Ω	Headlands near Dar Bouazza	−7, 49	33, 32
8	River *Ophiodes*	10, 00	20, 00	Ω, Ω	Oued Oum er-Rbia	−8, 20	33, 19
9	*Bagazei*	11, 00	19, 00	Ω, Ω	–	–	–
10	River *Nuios*	10, 00	18, 20	Ω, Ω	Oualidia lagoon	−9, 02	32, 44
11	*Saluentia* Capes	9, 30	17, 30	Ω, Ω	Cap Cantin and a southern headland	−9, 17	32, 33
12	River *Massa*	10, 30	16, 30	Ω, Ω	Oued Massa	−9, 40	30, 05
13	*Iarzeitha*	10, 00	15, 30	Ω, Ω	–	–	–
14	River *Daras*	10, 00	15, 00	Ω, Ω	Oued Draa	−11, 07	28, 41
15	Large Port	10, 00	14, 00	Ω, Ω	Khnifiss Lagoon	−12, 14	28, 03
16	*Babiba*	10, 30	13, 00	Ω, Ω	–	–	–
17	Cape *Arsinarion*	8, 00	12, 00	Ω, Ω	Cap Juby	−12, 55	27, 57
18	Cape *Rusaddir*	8, 30	11, 30	Ω, Ω	Ras Afkir Oum M'Bark	−13, 09	27, 42
19	River *Stacheir*	9, 30	11, 00	Ω, Ω	Wad As Saguia al Hamra	−13, 23	27, 11
20	Port of *Perphosios*	11, 00	10, 30	Ω, Ω	–	–	–
21	Cape *Katharon*	12, 30	9, 30	Ω, Ω	Cap Blanc	−17, 03	20, 46
22	River *Nias*	13, 30	9, 00	Ω, Ω	Senegal	−16, 31	15, 56
23	*Hesperu Keras*	13, 00	8, 00	Ω, Ω	Bight of Benin	3, 00	6, 20
24	River *Masitholos*	14, 00	6, 40	Ω, Ω	Niger and Benue	6, 04	4, 17
25	*Hypodromos Aithiopias*	14, 00	5, 15	Ω, Ω	–	–	–

direction (see e.g. Gyory et al., 2013). Ancient ships were driven by oar and sail; the speed was mainly influenced by the direction of the wind (Casson, 1971, 270–82). The predominant wind at the northern African western coast is the northeastern trade wind. Its influence in this region can reach up to 35° N (cf. Lockwood, 2005, p. 129, Fig. A79). A consideration of the current and the wind and of the speed of ancient journeys (Appendix A) shows that in ancient times the speed of a southward journey along the African western coast may have been about

$$v_{\mathrm{ns}} = 1300\,\mathrm{st\,day}^{-1} \tag{7}$$

and the speed of a northward voyage

$$v_{\mathrm{sn}} = 400\,\mathrm{st\,day}^{-1}. \tag{8}$$

For the localisation of Ptolemy's places, the distances between places are required. Assuming that the distance between two Ptolemaic positions (Λ_i, Φ_i) and (Λ_j, Φ_j) is based on a journey time, the reconstruction of this distance should take into account the underlying ancient conversion. The procedure applied is the stepwise conversion

$$\Lambda_i, \Phi_i, \Lambda_j, \Phi_j \xmapsto{1} \hat{s}(°) \xmapsto{2} \hat{s}(\mathrm{st}) \xmapsto{3} \Delta\hat{t} \xmapsto{4} \hat{s}(\mathrm{st}) \xmapsto{5} \hat{s}(\mathrm{m}) \tag{9}$$

(symbol $\xmapsto{x}$: step x of conversion). Since Ptolemy's procedure of the determination of geographic coordinates is unknown in individual cases, in step 1 the spherical distance (Eq. B1) is calculated between the two positions, which can be regarded as a sound approximation. In step 2, Eq. (1) is used. In step 3, the estimated ancient journey time is $\Delta\hat{t} = \hat{s}/v_3$. v_3 is the speed underlying the ancient conversion of a journey time into a distance. According to Sect. 3.2.2, the common ancient speed $v_{\varnothing}$ (Eq. 3) is an appropriate value, which is used. In step 4, the estimated actual distance is

Table 3. Places of Ptolemy's *Geography* at the African western coast as well as further inland places and their identifications, part 3. S.: source of Λ and Φ.

No.	Ancient name	Λ (°, ′)	Φ (°, ′)	S. Λ, Φ	Identification	λ (°, ′)	ϕ (°, ′)
GH 4.6 *Libya Interior* – further places							
26	*Mandron* Mtns., middle	14, 00	19, 00	Ω, Ω	Middle, High and Anti-Atlas, middle	−7, 10	31, 31
27	*Sagapola* Mtns., middle	20, 20	22, 00	Ω, Ω	In the Middle Atlas	−4.55	33, 31
28	*Rhussadion* Mtns., middle	17, 00	11, 00	Ω, Ω	Southwest of Jebel Ouarkziz	−10, 43	27, 40
29	*Ochema Theon* Mtn.	19, 00	5, 00	Ω, Ω	Mount Cameroon and Western High Plateau	10, 30	6, 30
30	*Kapha* Mtns., middle	27, 00	10, 30	Ω, X	(See Sect. 4.6)	–	–
48	*Autolalai*	10, 00	23, 50	Ω, Ω	–	–	–
55	*Malachath*	20, 20	20, 15	Ω, Ω	–	–	–
57	*Byntha*	20, 15	21, 00	Ω, Ω	–	–	–
94	Island of *Kerne*	5, 00	25, 40	Ω, Ω	Herne	−15, 47	23, 52
95	Island of *Hera* or *Autolala*	8, 00	23, 50	Ω, Ω	Mogador	−9, 47	31, 30
96	Island of *Aprositos*	1, 00	16, 00	Ω, Ω	Alegranza	−13, 31	29, 24
97	Island of *Juno*	1, 00	15, 15	Ω, Ω	Graciosa	−13, 31	29, 15
98	Island of *Pluvialia*	1, 00	14, 15	Ω, Ω	Lanzarote	−13, 38	29, 02
99	Island of *Capraria*	1, 00	12, 30	Ω, Ω	Fuerteventura	−14, 02	28, 22
100	Island of *Canaria*	1, 00	11, 00	Ω, Ω	Gran Canaria	−15, 36	27, 58
101	Island of *Centuria* (*Pintuaria*)	1, 00	10, 30	Ω, Ω	Tenerife	−16, 35	28, 17

$\hat{s} = v_4\,\Delta\hat{t}$. v_4 is the actual speed of the ancient journey for which v_{ns} (Eq. 7) is applied in the area of the Canary Current and the trade wind. In step 5, the Italian and Egyptian stade (Eqs. 5 and 6) are used.

A special case of Eq. (9) is the equality of the ancient assumed speed v_3 and the actual ancient speed v_4, e.g. $v_3 = v_4 = v_\varnothing$. Then, the conversion corresponds to the direct conversion

$$\Lambda_i, \Phi_i, \Lambda_j, \Phi_j \xmapsto{1} \hat{s}(^\circ) \xmapsto{2} \hat{s}(\mathrm{st}) \xmapsto{5} \hat{s}(\mathrm{m}) \qquad (10)$$

of Ptolemy's distance into a metric distance. Conversion Eq. (10) is applied to distances outside the area of the Canary Current and the trade wind and to inland distances.

If further coastal places are given by Ptolemy between the two coastal sites i and j being considered, the sum

$$\hat{s}_i^j = \sum_{k=i}^{j-1} \hat{s}_{k,k+1} \qquad (11)$$

of the single distances $\hat{s}$ of the intermediate places has to be determined (order of the indices = order of the places).

The assumptions underlying the conversions according to Eqs. (9) and (10) may be wrong. Therefore, a correction factor a is applied to distances $\hat{s}$ if they reveal an obvious distortion. Then, the calculated distance becomes

$$\overline{s}(a) = a\hat{s}. \qquad (12)$$

Factor a is determined from the data.

$\overline{s}_{i,j}$ or $\overline{s}_i^j$ can be compared with the actual distance s. In the case of two inland places, the spherical distance is determined; in the case of coastal places, the path length along the coast is measured. The determination of actual distances was carried out by means of the Google Earth application (Google Inc., 2015).

3.2 Accuracy of Ptolemy's distances

The distances derived from Ptolemy's coordinates are possibly adulterated by diverse errors, such as rounding errors of the distances and coordinates and unsuitable conversions of journey times into distances. In the following, possible errors are considered (Sect. 3.2.1), and Ptolemy's distances are compared with distances given by Pliny in order to gain an insight into the reliability of the data (Sect. 3.2.2).

3.2.1 Errors of distances

The Ptolemaic coordinates Λ and Φ are rounded values. A few latitudes in GH 4.1 have a fraction of degree of $\frac{11}{12}$ so that Ptolemy used a resolution of $\frac{1}{12}{}^\circ$ in these cases. The majority of the coordinate values surely has a rougher resolution (cf. Marx, 2011). An investigation of the propagation of the rounding errors of coordinates (Appendix B) shows that generally they may have caused distance errors up to 10 km (GH 4.1) or 30 km (GH 4.6), respectively.

Several large, partly systematic errors of the Ptolemaic distances (see Sect. 5) are not explicable by rounding errors. The following types of adulterations of distances may have occurred.

1. Distances were altered through a rough specification of journey times or the use of the measurement units D and DN, respectively (Sect. 2). If the most precise reso-

Table 4. Comparison of Pliny's distances s_N and Ptolemy's distances $\hat{s}$ derived from his coordinates by Eq. (9), steps 1 and 2, and Eq. (4). T.: type of $\hat{s}$; d: direct spherical distance Eq. (B1); s: sum Eq. (11).

Pliny				Ptolemy			
From – to	s_N (Rmi)	s_N (st)	s_N (DN)	From – to	T.	$\hat{s}$ (st)	$\hat{s}$ (DN)
1. NH 5.1; Pliny/natives; 1 DN = 800 st				GH 4.1			
Lixos (town) – *Sububus*	50	400	0.5	47 *Lix* – 4 *Subur*	d	450	0.5
Sububus – *Sala*	50	400	0.5	4 *Subur* – 7 *Sala*	s	530	0.5
Salat – *Asana*	150	1200	1.5	6 *Sala* – 12 *Asana*	s	1362	≈ 1.5
Fut – *Atlas*	200	1600	2.0	16 *Phthuth* – 24 Bigger *Atlas*	s	2547	2.5
2. NH 5.1; Agrippa; 1 DN = 112 Rmi = 896 st							
Lixos (river) – *Anatis*	205	(?)	≐2	3 *Lix* – 12 *Asana*	s	1918	≈ 2
Lixos (river) – Straits of *Gades*	112	896	1.0	–	–	–	–
Lixos (river) – *Rutubis*	224	1792	2.0	3 *Lix* – 11 *Rusibis*	s	1631	≈ 2
Extent of a gulf	616	4928	5.5	–	–	–	–
3. NH 6.37				GH 4.6			
Hesperian Prom. – *Theon Ochema*	–	–	4	23 *Hesperu Keras* – 29 *Ochema Theon*	s	4032	4.0
4. NH 6.37; Sebosus, Juba; 1 DN = 1000 st							
Gades – *Junonia*	750	6000	6.0	*Gades* (GH 2.4) – 95 *Hera/Autolala*	d	6287	≈ 6
Junonia – *Pluvialia*	750	6000	6.0	95 *Hera/Autolala* – 98 *Pluvialia*	d	5820	≈ 6
Pluvialia – *Invallis*	250	2000	2.0	98 *Pluvialia* – 101 *Centuria*	d	1875	≈ 2
Purple Islands – Fortunate Islands	625	5000	5.0	95 *Hera/Autolala* – 96 *Aprositos*	d	5113	≈ 5

lution was $\frac{1}{2}$ D, errors up to $\frac{1}{4}$ D were possible, i.e. about 20 km.

2. The speed of a sea voyage was influenced by currents and winds (Sect. 3.1), which was not taken into account in the conversion of a journey time into a distance. (a) If the assumed speed of a northward voyage was too large, the distance became too large. (b) If the assumed speed of a southward voyage was too small, the distance became too small. If a journey had the length s and if the actual speeds were v_{ns} (Eq. 7) and v_{sn} (Eq. 8), respectively, and provided that $v_{\varnothing}$ (Eq. 3) was used for the conversion of the journey time into a distance s', then it follows that $s' = v_{\varnothing}/v_{\mathrm{ns}}\, s = 0.77\, s$ and $s' = v_{\varnothing}/v_{\mathrm{sn}}\, s = 2.5\, s$. Consequently, the distances are significantly underestimated and overestimated.

3. The measurement units D and DN were confused because, for example, the distance was noted improperly as an x-day journey. (a) If a distance based on x D was mistaken for x DN, the distance became too large. (b) If a distance based on x DN was mistaken for x D, it became too small.

3.2.2 Comparison with Pliny's distances

Pliny gives several distances between locations at the African western coast, see Table 4 (based on Winkler and König, 1993; Brodersen, 1996), which can be compared with Ptolemy's distances. Müller (1902) and Spaul (1958, p. 6.2) make comparisons for the distances of NH 6.37 based on st and for the distances of NH 5.1 based on Rmi, respectively, revealing more or less significant differences from Ptolemy's distances. Müller (1902), however, remarks that the distances are rough estimates based on journey times. This is taken into account in the following comparison.

Table 4 provides the sources of Pliny's information, Pliny's distances s_N in Rmi and st (1 Rmi = 8 st) and the assumed original values in DN. The distances are grouped according to the assumed conversion between the units DN and st.

In group 1, the distances are multiples of 50 Rmi or of 400 st, so that they may be based on 1 DN = 800 st. The resulting distances are multiples of $\frac{1}{2}$ DN and may be the original values. The distances of group 2 probably originate with Agrippa (Marcus Vipsanius Agrippa, ca. 64–12 BC; cf. translation Winkler and König, 1993). With one exception, these distances are multiples of 56 Rmi (Klotz, 1931) or of 448 st. Thus, the ancient conversion may be based on 1 DN = 900 st ≈ 112 Rmi. The distances of group 4 are multiples of 125 Rmi or 1000 st; they are probably based on 1 DN = 1000 st.

Table 4 also gives Ptolemy's distances $\hat{s}$ in st (Eq. 9, steps 1 and 2). (The equations of Ptolemy's with Pliny's places are based on Forbiger (1844), Spaul (1958) and Sects. 4 and 6.)

In the case of $\hat{s}_{47.4}$, the X variant is used, in the other cases Ω (*Gades*, GH 2.4.16: $\Lambda = 5°10'$, $\Phi = 36°10'$). The (M9) Smaller *Atlas* is not included in the sum Eq. (11) because it does not exist at the coast (see Sect. 4.1) and was probably inserted by Ptolemy.

Pliny's and Ptolemy's distances can be used for an estimation of the relation between the units st and DN that underlies Ptolemy's distances. To do so, for each Ptolemaic distance $\hat{s}$ (in st) of Table 4 the observation equation $\hat{s}_i + v_i = c\, s_{\mathrm{N}i}$ is set up, where $i = 1 \ldots 11$ is the index of the distance, $s_{\mathrm{N}i}$ is Pliny's distance expressed in DN, c is an unknown factor in $\mathrm{st\,DN^{-1}}$, and v_i is a presumably random residual. c is determined by a least squares adjustment (see e.g. Böck, 1961; Baumann, 1993, 17–20); the result is $1008\,\mathrm{st\,DN^{-1}} \pm 21\,\mathrm{st\,DN^{-1}}$. Hence, the usual ancient relation $1\,\mathrm{DN} = 1000\,\mathrm{st}$ (Eq. 4) can be assumed for Ptolemy's distances. Table 4 shows $\hat{s}$ expressed in DN. It reveals that Pliny's and Ptolemy's distances are identical in most cases. A few distances are reviewed in the following.

Ptolemy's distance *Fut–Atlas* probably refers to the Anti-Atlas; see Sect. 4.3; Pliny, however, may refer to the High Atlas, so that his distance is shorter.

Pliny's distance *Lixos–Anatis* cannot be derived from a journey time as the other distances. Spaul (1958, p. 6.2) assumes that 205 = CCV is a corruption of 250 = CCL. This is possible because the sum of Plinus' three distances *Lixos–Sububus–Sala*/*Salat–Asana* (= *Anatis*) also amounts to 250 Rmi (the River *Salat* is near the town of *Sala*).

The distance Hesperian Promontory–*Theon Ochema* is given directly by Pliny; it is also to be found in *Peripl.* 16. (Pliny gives, referring to Agrippa, a differing value of 10 DN in NH 5.1. Since, however, Agrippa places the *Atlas* in the middle of this distance, the information seems to be unreliable and is not considered here.) Ptolemy's distance is calculated over the waypoints nos. L23–L25 and L29 and equals nearly exactly Pliny's distance.

Consequently, the investigation shows that the ancient distance data are consistent.

4 Ptolemy's places at the African western coast

In the following, the places of GH 4.1 *Mauritania Tingitana* (Sects. 4.1–4.3) and GH 4.6 *Libya Interior* (Sects. 4.5–4.7) at the African western coast as well as further places are localised. In Sect. 4.4 the repetition of places of GH 4.1 in GH 4.6 is investigated.

Tables 1–3 list the modern counterparts of the Ptolemaic places considered and their geographic longitude λ (relative to Greenwich) and latitude ϕ; Fig. 3 shows the northern positions (no. M5 is omitted). Tables 5 and 6 give the localisations based on distances including these distances. Due to uncertainties and missing evidence, not all places can be localised.

The sequence of the Ptolemaic places in the following sections corresponds to that of their identification. Their modern counterparts are indicated by bold text. Unless otherwise stated, Pliny's information originates from NH 5.1.

4.1 *Mauritania Tingitana* part 1: up to the *Asana*

Firstly, the correction Eq. (12) is not used for distances $\hat{s}$. The (9) Smaller *Atlas* is not included in the sum Eq. (11) of distances; cf. Sect. 3.2.2 and no. 9 below.

(1) Cape *Kotes*, (3) River *Lix*, (4) River *Subur*, (7) *Sala*, (47) *Lix*, (51) *Tamusida*, (57) *Volubilis*: these places are either known or consistently identified in the literature (e.g. in Spaul, 1958; S & G).

(43) *Diur* (mountains), middle: Spaul (1958, p. 8.8): Djebel Zerhoun; Lacroix (1998, p. 254): Rif. The Djebel Zerhoun is situated south of (57) *Volubilis*/Moulay Idriss, which contradicts Ptolemy's position northeast of it. The *Diur* are the only mountains in the north of *Mauritania Tingitana* and are located between (57) *Volubilis* and (34) *Akrath*; the latter is situated at the northern coast (Fig. 1). This applies to the **Rif**.

(46) *Zilia*: Mannert (1825, p. 467), Forbiger (1844, p. 878, fn. 14), S & G: Asilah; Spaul (1958, p. 7.25), Aujac et al. (1969–2015, vol. 15, p. 80), Talbert (2000): Dchar Jedid. The identification of *Zilia* with **Dchar Jedid** 12 km northeast of Asilah is substantiated by inscriptions naming the town *Zilil* or *Zili(s)* (e.g. Aujac et al., 1969–2015). Λ of X fits better with that of (3) *Zilia* (Ω: no. 46 6°30′).

(2) River *Zilia*: Mannert (1825, p. 468), Forbiger (1844, p. 869, fn. 82): small river at Asilah; Spaul (1958, p. 6.5): Oued Mharhar; Talbert (2000): Oued Khobs. Owing to the short distance $\hat{s}_{46.2}$ of X (Ω: no. 2 $\Phi = 35°40'$) and owing to its name, River *Zilia* may be near (46) *Zilia*/Dchar Jedid as in the cases of (6) River *Sala* and (7) *Sala* as well as (3) River *Lix* and (47) *Lix*. This is met by the **Oued el Gharifa**, whose mouth is west of Dchar Jedid. Ruins and archaeological findings suggest a port at the mouth in the first century BC (Euzennat, 1976).

(6) River *Sala*: Mannert (1825, p. 472), Forbiger (1844, p. 868), Spaul (1958, p. 6.9), Talbert (2000), S & G: Oued Bou Regreg. The *Sala* is situated between the (4) *Subur*/Oued Sebou and (7) *Sala*/Chellah. There is only the **Oued Bou Regreg**. Λ of X is in better agreement with Λ of (7) *Sala* and Λ of (51) *Tamusida* than Λ of Ω (6°10′).

(5) *Emporikos kolpos*: Ptolemy places this bay between the (4) *Subur* and the (6) *Sala*, i.e. **between the Oued Sebou and the Oued Bou Regreg**. It has been assumed that it corresponds to the bay of *Sagigi* mentioned by Pliny (e.g. Spaul, 1958, p. 6.3; S &G , p. 383, fn. 9). According to Pliny (translation Winkler and König, 1993), the bay of *Sagigi* is situated between the (3) *Lixos* (*Lix*/Uad Lucus) and *Mulelacha*, which is further north than the *Sububa* (*Subur*) and is assumed to be Moulay Bou Selham (e.g. Winkler and König, 1993, p. 120). Thus, there is a distance of at least 80 km between the two

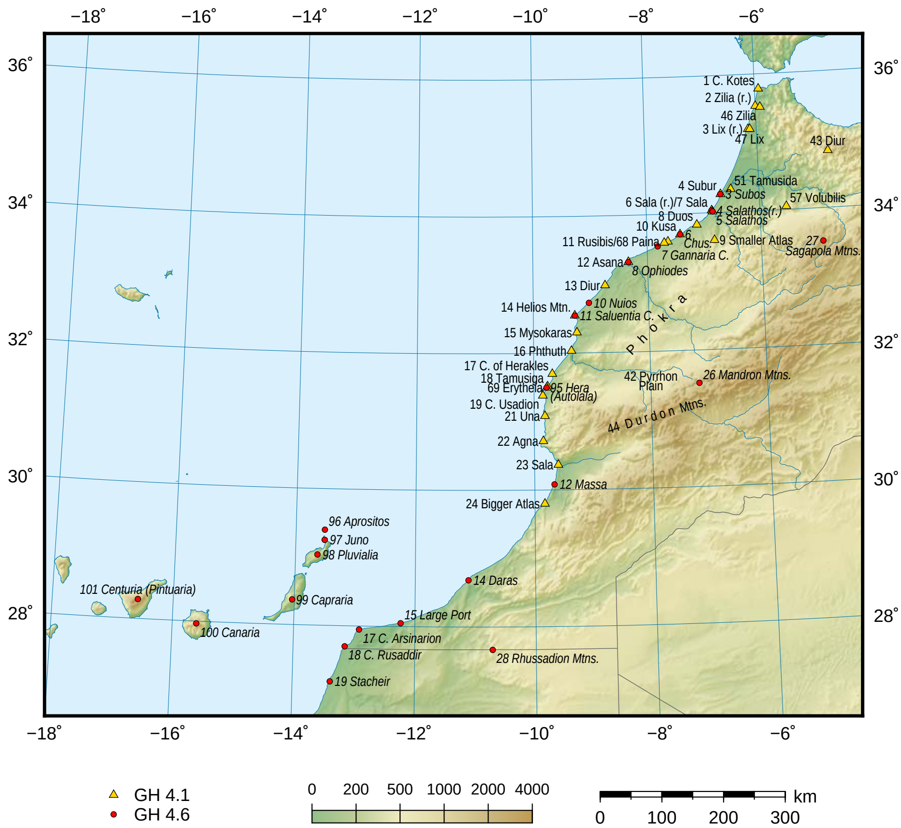

Figure 3. Identified northern places in GH 4.1 *Mauritania Tingitana* (upright) and GH 4.6 *Libya Interior* (italic).

bays so that they do not seem to be identical. Alternatively, Ptolemy's or Pliny's information may either be erroneous or both refer to a bay which reaches approximately from the Uad Lucus (*Lix*) to the Oued Bou Regreg (*Sala*). If Ptolemy refers to a coastal point, Λ of X is to be preferred in agreement with no. 6 (Ω: 6°10′).

(12) River *Asana*: Mannert (1825, p. 473), Forbiger (1844, p. 869), S & G: Oued Oum er-Rbia. The rivers closest to $\hat{s}_7^{12}$ are the Oued Oum er-Rbia and the Oued Tensift ($s \doteq 160$ km, 350 km). The **Oued Oum er-Rbia** is chosen in agreement with the mentioned literature. Forbiger (1844) states that the ancient name of the river can be recognised in the name of the town Azamur (Azemmour) at the river mouth.

From the ratio of $\hat{s}_7^{12}$ and the corresponding s follows the correction factor

$$a_1 = 0.654, \tag{13}$$

which is the average of the results from Ist and Est. a_1 is applied in Eq. (12) up to no. 12.

(8) River *Duos*: Spaul (1958, p. 6.11): Oued Cherrat; S & G: Oued Mellah. $\bar{s}_{7,8}(a_1)$ of Ω (no. 8 $\Phi = 33°20'$) is met by the Oued Mellah and the Oued Nefifikh ($s \doteq 60$ km), but then there would be no large river which satisfies the position of the (10) *Kusa* between the (8) *Duos* and the (12) *Asana*. Therefore, the **Oued Cherrat** is chosen, which is in accordance with $\bar{s}_{7,8}(a_1)$ of X.

(10) River *Kusa*: Spaul (1958, p. 6.11): Oued Nefifikh; S & G: Oued Sous. For the (8) *Duos*, Φ of X has to be applied; see no. 8 above. The Oued Mellah and the Oued Oum er-Rbia are the rivers closest to $\bar{s}_{8,10}(a_1)$ ($s \doteq 33$ km, 130 km). Since the latter is equated with the (12) *Asana*, the *Kusa* is the **Oued Mellah**.

(9) Smaller *Atlas* Mountain: between the (8) *Duou*/Oued Cherrat and the (10) *Kusa*/Oued Mellah there are no mountains at the coast or in its vicinity. South-east of the mouth of the Oued Cherrat, however, parts of the **Moroccan Meseta west of the Middle Atlas** are close to the coast. Ptolemy probably features these high plains and mountains wrongly as a coastal point far to the west.

(11) Port *Rusibis*: Gosselin (1798–1813, p. 125), Mannert (1825, p. 473), S & G: Mazagan (El Jadida, El-Bridja); Spaul (1958, p. 6.16): Azemmour; Lacroix (1998, p. 170): Casablanca. Pliny gives 224 Rmi for the distance (3) *Lix*/Uad Lucus–(11) *Rusibis* and 205 Rmi for (3) *Lix*–(12) *Asana*/Oum er-Rbia. Hence, *Rusibis* would be further south than the *Asana*. This assumption, however, is not necessary. First, Pliny's distances contradict Ptolemy's positioning of the *Asana* south-east of *Rusibis*. Second, Pliny's distance of 205 Rmi is indeed correct, but its high accuracy is

Table 5. Localisation of the places of GH 4.1 *Mauritania Tingitana*. S.: source of the Ptolemaic position; a: correction factor; $\overline{s}(a)$: corrected Ptolemaic distance Eq. (12); s: actual distance; Ist, Est: based on the Italian/Egyptian stade.

Place wanted	Distance from	S.	a	$\overline{s}(a)$ (km)		Identification	s
				Ist	Est		(km)
s equals the length of the sea route; $\hat{s}$ by Eq. (9); $v_4 = v_{ns}$							
8 *Duos*	7 *Sala*/Chellah	Ω	a_1	51	44	Oued Cherrat	33
		X	a_1	35	30		
10 *Kusa*	8 *Duos*/Oued Cherrat	Ω	a_1	57	48	Oued Mellah	33
		X	a_1	79	67		
11 Port *Rusibis*	10 *Kusa*/Oued Mellah	Ω	a_1	20	17	Casablanca	23
12 *Asana*	7 *Sala*/Chellah	Ω	1	264	225	Oued Oum er-Rbia	161
13 *Diur*	12 *Asana*/Oued Oum er-Rbia	Ω	a_2	31	26	Sidi Moussa lagoon	60
14 *Helios* Mtn.	13 *Diur*/Sidi Moussa lagoon	Ω	a_2	46	39	Cap Cantin	69
15 *Mysokaras*	14 *Helios* mtn./Cap Cantin	Ω	a_2	46	39	Safi	31
	16 *Phthuth*/Oued Tensift	Ω	a_2	26	22		32
		X	a_2	43	36		
16 *Phthuth*	14 *Helios* Mtn./Cap Cantin	Ω	a_2	72	61	Oued Tensift	61
		X	a_2	89	75		
17 C. of Herakles	16 *Phthuth*/Oued Tensift	Ω	a_2	35	30	Cap Hadid	49
18 *Tamusiga*	17 C. of Herakles/Cap Hadid	Ω	a_3	25	21	Essaouira	23
19 C. *Usadion*	17 C. of Herakles/Cap Hadid	Ω	a_3	70	59	Cap Sim	40
21 *Una*	19 C. *Usadion*/Cap Sim	Ω	a_3	57	49	Oued Iguezoullene	36
22 *Agna*	21 *Una*/Oued Iguezoullene	Ω	a_3	50	43	Oued Tamri	43
23 *Sala*	22 *Agna*/Oued Tamri	Ω	1	70	59	Oued Sous	53
24 Bigger *Atlas*	23 *Sala*/Oued Sous	Ω	1	107	91	Foothills, Anti-Atlas (northern end)	79
s equals the spherical distance; $\hat{s}$ by Eq. (10)							
2 *Zilia* (river)	46 *Zilia*/Dchar Jedid	Ω	1	60	51	Oued el Gharifa	8
		X	1	20	17		
42 *Pyrrhon* Plain	17 C. Herakles/Cap Hadid	Ω	1	160	136	Haouz plain	160
44 *Durdon*	19 C. *Usadion*/Cap Sim	Ω	1	203	173	High Atlas, Toubkal	186

Table 6. Localisation of the places of GH 4.6 *Libya Interior*. S.: source of the Ptolemaic position; a: correction factor; $\overline{s}(a)$: corrected Ptolemaic distance Eq. (12); s: actual length of the sea route; Ist, Est: based on the Italian/Egyptian stade.

Place wanted	Distance from	S.	a	$\overline{s}(a)$ (km)		Identification	s
				Ist	Est		(km)
$\hat{s}$ by Eq. (9), $v_4 = v_{ns}$							
7 *Gannaria* C.	6 *Chusarios*/Oued Mellah	Ω	a_4	47	40	C. near Dar Bouazza	43
8 *Ophiodes*	7 *Gannaria* C./c. at Dar Bouazza	Ω	a_4	26	22	Oued Oum er-Rbia	55
10 *Nuios*	8 *Ophiodes*/Oum er-Rbia	Ω	a_4	95	81	Oualidia lagoon	96
	11 *Saluentia* C./Cap Cantin	Ω	a_4	36	31	Oualidia lagoon	32
12 *Massa*	11 *Saluentia* C./Cap Cantin	Ω	1	167	142	Oued Massa	305
14 *Daras*	12 *Massa*/Oued Massa	Ω	1	194	165	Oued Draa	212
15 Large Port	14 *Daras*/Oued Draa	Ω	1	120	102	Khnifiss Lagoon	132
17 C. *Arsinarion*	14 *Daras*/Oued Draa	Ω	1	572	486	Cap Juby	200
18 C. *Rusaddir*	17 C. *Arsinarion*/Cap Juby	Ω	1	84	72	Ras Afkir Oum M'Bark	39
		X	1	190	162		
19 *Stacheir*	18 C. *Rusaddir*/Ras Afkir Oum M'Bark	Ω	1	133	113	Wad As Saguia al Hamra	61
		X	1	118	100		
$\hat{s}$ by Eq. (10)							
23 *Hesperu Keras*	29 *Ochema Theon*/Mount Cameroon	Ω	1	740	630	Bight of Benin (middle)	750

very unlikely in view of that of the other ancient data. Pliny's two distances may originate from rough specifications as 2 DN; see Sect. 3.2.2. Third, 205 Rmi may be a corruption of 250; three other distances given by Pliny yield the sum of 250 Rmi for this distance; see Sect. 3.2.2. In contrast to the usual identification Mazagan, **Casablanca** is consistent with Ptolemy's positioning and also with his distance $\bar{s}_{10,11}(a_1)$.

(68) Island of *Paina*: Φ of *Paina* equals that of the (12) *Asana*/Oum Er-Rbia, and the island is positioned at a large distance to the coast. This is met by the Madeira archipelago. Ptolemy, however, often positions islands much too far from the coast (see e.g. England and Italy in Kleineberg et al., 2012; Marx and Kleineberg, 2012). Owing to its Φ, *Paina* is probably situated near the (12) *Asana*/Oum Er-Rbia. There exists the **island of Sidi Abderrahmane**, which is only 10′ more northern than the river mouth.

4.2 *Mauritania Tingitana* part 2: up to the Cape of Herakles

The sum $\hat{s}_{12}^{24}$ from the *Asana*/Oued Oum er-Rbia to the Bigger *Atlas* amounts to 866/737 km (Ist/Est). The Bigger *Atlas* must correspond to the foothills of the High Atlas or of the Anti-Atlas near the coast. The resulting actual distances are significantly smaller than $\hat{s}_{12}^{24}$. Since the Anti-Atlas yields a smaller difference, it is preferred to the High Atlas. (Gosselin (1798–1813), Spaul (1958) and Lacroix (1998) also assume that the Bigger *Atlas* is further south than the High Atlas.) The foothills of the Anti-Atlas (at 29°48′ N) begin at $s \doteq 472$ km. The ratio $s/\hat{s}$ yields the correction factor

$$a_2 = 0.588 \tag{14}$$

(average based on Ist and Est), which is used in Eq. (12) in the following.

(13) River *Diur*: Spaul (1958, p. 6.12): Oualidia lagoon. At $\bar{s}_{12,13}(a_2)$ there is no river; the nearest river is the Oued Tensift at $s \doteq 190$ km. Possibly, the *Diur* refers to a lagoon, which was taken for a river. The Oualidia lagoon is situated at $s \doteq 97$ km, but the **Sidi Moussa lagoon** at $s \doteq 60$ km fits better with $\bar{s}_{12,13}(a_2)$.

(14) *Helios* Mountain: Mannert (1825, p. 475), Forbiger (1844, p. 867), Spaul (1958, p. 6.10), S & G: Cap Cantin (Beddouza); Talbert (2000, *Soloeis*): Cap Spartel. The cape *Helios* Mountain is also named *Soloeis* (S & G, p. 383, fn. 12; *Peripl.* 3). The only cape in the vicinity of $\bar{s}_{13,14}(a_2)$ is **Cap Cantin**.

(16) River *Phthuth*: Gosselin (1798–1813, p. 125), Mannert (1825, p. 476), Forbiger (1844, p. 869), Spaul (1958, p. 6.16): Oued Tensift. The **Oued Tensift** is situated in accordance with the lower values of $\bar{s}_{14}^{16}(a_2)$, which is followed by only a few short rivers. Φ of Ω fits better than Φ of X (30°15′).

(15) Port *Mysokaras*: Gosselin (1798–1813, p. 125), Mannert (1825, p. 476), Spaul (1958, p. 6.14): Safi. Today, **Safi** is the largest town between (14) *Helios* Mountain/Cap Cantin and the (16) *Phthuth*/Oued Tensift. It is consistent with $\bar{s}_{14,15}(a_2)$ and $\bar{s}_{15,16}(a_2)$ (Ω; see no. 16 above).

(17) Cape of Herakles: Mannert (1825, p. 476), Forbiger (1844, p. 867), Spaul (1958, p. 6.13): cape/peninsula of Mogador; S & G: Cap Hadid. The only large cape in the vicinity of $\bar{s}_{16,17}(a_2)$ (Ω; see no. 16 above) is **Cap Hadid**. The cape is marked by the nearby mountain of Jebel Hadid, which is visible from the sea (cf. Arlett, 1836, p. 289).

(42) *Pyrrhon* Plain: Spaul (1958, p. 7.19): Haouz plain; Talbert (2000): possibly Gharb plain. The *Pyrrhon* Plain is east of the (17) Cape of Herakles/Cap Hadid. The **Haouz plain** is almost east of Cap Hadid and in accordance with $\hat{s}_{17,42}$.

4.3 *Mauritania Tingitana* part 3: up to the Bigger *Atlas*

The sum $\hat{s}_{17}^{24}$ from the Cape of Herakles/Cap Hadid to the Bigger *Atlas* is 553/470 km (Ist/Est), but the actual distance to the foothills of the Anti-Atlas (cf. Sect. 4.2) is only $s \doteq 240$ km. This yields the new correction factor

$$a_3 = 0.469 \tag{15}$$

(average based on Ist and Est), which is used in Eq. (12).

(18) *Tamusiga*: Gosselin (1798–1813, p. 125): at the Oued Ksob (near Essaouira); Mannert (1825, p. 476), S & G: Mogador (Essaouira). **Essaouira** is consistent with $\bar{s}_{17,18}(a_3)$. Excavations revealed that there was a Phoenician settlement in the seventh century BC (Culican, 1991, p. 543).

(19) Cape *Usadion*: Gosselin (1798–1813, p. 125), Spaul (1958, p. 6.13): Cap Ghir; Mannert (1825, p. 476), Forbiger (1844, p. 867), S & G (p. 385, fn. 13): Cap Sim (Osem). In the vicinity of $\bar{s}_{17}^{19}(a_3)$ there are Cap Sim and Cap Tafelney ($s \doteq 40$ km, 73 km). The latter would cause a lack of space for the places up to the Bigger *Atlas*, so that **Cap Sim** is chosen.

Mountain *Phokra*: the *Phokra* reaches from the (9) Smaller *Atlas*/Moroccan Meseta west of the Middle Atlas to (19) Cape *Usadion*/Cap Sim (GH 4.1.12). In this region the Moroccan Meseta is situated at some distance from the coast so that the *Phokra* corresponds to the **southern part of the Moroccan Meseta**.

(69) Island of *Erytheia*: Gosselin (1798–1813, p. 160), Mannert (1825, p. 476): Mogador. The large longitudinal distance of the island to the coastal places is of no account; see Sect. 4.1, no. 68. *Erytheia* is situated about 1° further south than (18) *Tamusiga*/Essaouira, where no island exists. At Essaouira, however, there is **Mogador**. According to Pliny (NH 6.36), Juba (Juba II, king of Mauritania) discovered a few islands in the latitudes of the Autololes, where he established the production of Gaetulian Purple (Purple Islands). Pliny (NH 5.1) locates the Autololes south of (7) *Sala*/Chellah and north of the mountain *Atlas*, which probably refers to the High Atlas (see also Sect. 3.2.2). Furthermore, according to Agrippa's description of the coast given by Pliny, the Autololes are south of the promontory

Soloeis and north of the River *Masathat*, which probably correspond to Cap Cantin and the Oued Massa (e.g. Winkler and König, 1993, p. 120) as well as to Ptolemy's (14) *Helios* Mountain and (L12) *Massa* (see Sect. 4.2, no. 14; Sect. 4.6, no. 12). In conclusion, the Autololes were situated between Cap Cantin and the High Atlas. The only islands in this region are at Essaouira (Mogador). Thus, these are the Purple Islands, and *Erytheia* refers to Mogador, the largest of them. It has been suggested that the Purple Islands are the Canary Islands (e.g. Hennig, 1944, p. 45), where orchil lichen may have been used for dyeing. Müller (1902), however, points out that Pomponius Mela and Pliny mention seashells as the origin of the Gaetulian Purple (Mela: "Those coasts [of the Nigritae and the Gaetuli] are very famous for purple and murex – the most effective dyeing materials", *Chorography* 3.104, Romer, 1998; Pliny: "… all the rocks of Gaetulia are searched for the murex and the purple").

(21) River *Una*: Gosselin (1798–1813, p. 125): Oued Sous; Mannert (1825, p. 476): Oued Iguezoullene (Iguzul); Spaul (1958, p. 6.15): Oued Massa. The only large river in the vicinity of $\bar{s}^{21}_{19}(a_3)$ is the **Oued Iguezoullene**.

(22) River *Agna*: Spaul (1958, p. 6.15): Oued Adoudou; S & G: Oued Tamri (Beni-Tamer). The largest river in the vicinity of $\bar{s}_{21,22}(a_3)$ is the **Oued Tamri**.

(23) River *Sala*: Mannert (1825, p. 477): Oued Tamri (Beni-Tamer); Spaul (1958, p. 6.15): Oued Noun; S & G: Oued Tamraght. There are only very small rivers at $\bar{s}_{22,23}(a_3)$. The **Oued Sous**, however, is consistent with $\hat{s}_{22,23}$ (without factor a).

(24) Bigger *Atlas* Mountain: Gosselin (1798–1813, p. 125), Spaul (1958, p. 6.13): Cap Noun/Cap Draa; Lacroix (1998, p. 175): Anti-Atlas; S & G (p. 385, fn. 14): Cap Ghir. Owing to the fitting distance $\hat{s}_{22,23}$ (see no. 23 above), no correction factor a is applied to $\hat{s}_{23,24}$. Approximately at this distance the **foothills of the Anti-Atlas** begin.

(44) *Durdon* Mountains, western part: Spaul (1958, p. 8.8): Toubkal; Lacroix (1998, p. 166): High Atlas; S & G: Sidi-bel-Abbes. The *Durdon* Mountains are almost east of (19) Cape *Usadion*/Cap Sim. In an easterly direction and at a distance of $\hat{s}_{19,44}$, the **western parts of the High Atlas** are situated. Approximately at $\hat{s}_{19,44}$, **Toubkal** is located, which is the highest peak in the Atlas Mountains.

4.4 Identical places

If Ptolemy used different data sources for GH 4.1 *Mauritania Tingitana* and GH 4.6 *Libya Interior*, which contained different place names and distances, he might have not noticed identical places. Gosselin (1798–1813, p. 129) assumes that the following four coastal places of GH 4.1 also appear in GH 4.6 with similar names and in the same order:

- (M4) River *Subur* and (L3) River *Subos*,
- (M6) River *Sala* and (L4) River *Salathos*,

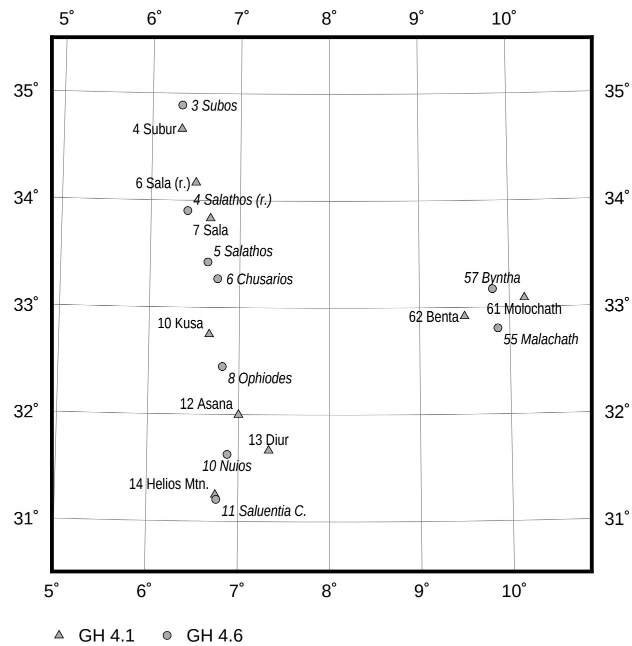

Figure 4. Presumably identical places of GH 4.1 *Mauritania Tingitana* (upright) and GH 4.6 *Libya Interior* (italic). The positions of GH 4.6 are geometrically transformed by means of an adjustment of a transformation of coordinates.

- (M7) *Sala* and (L5) *Salathos*,
- (M10) River *Kusa* and (L6) River *Chusarios*.

Gosselin (1798–1813) assumes a few further repetitions of coastal places. Since no explanations are obvious for them, they are not considered here (also declined by Pinkerton, 1817, p. 702, fn. *).

There are three further pairs of places which have similar names in GH 4.1 and GH 4.6 and similar relative positions with respect to the assumed duplicated places:

- (M61) *Molochath* and (L55) *Malachath*,
- (M62) *Benta* and (L57) *Byntha*,
- (M14) *Helios* Mountain (*Soloeis*) and (L11) *Saluentia* Capes.

The first two pairs are inland towns. Nos. M14 and L11 are both equated with the promontory *Soloeis* of *Peripl.* 3 (e.g. Forbiger, 1844, p. 867; Gosselin, 1798–1813, p. 130).

Consequently, identity is assumed for seven pairs of places here. This hypothesis is extended at the end of Sect. 4.5. The similar positions of the presumably duplicated places are illustrated in Fig. 4 by means of a transformation of coordinates (see below), which positions the places of GH 4.6 in the vicinity of the places of GH 4.1.

A repetition of the places of GH 4.1 in GH 4.6 leads to the following assumptions. The positions in GH 4.6 are arranged south of the (M24) Bigger *Atlas* so that in GH 4.6

the coordinates are shifted. Additionally, differences in the scale may exist between GH 4.1 and GH 4.6, for example, because their data sources are based on different determinations and conversions of journey times and because Ptolemy scaled distances. Furthermore, the directions of the identical stretches of the coast may differ so that a small rotation is present. The described systematic differences of coordinates between GH 4.1 and GH 4.6 are modelled by means of a two-dimensional transformation of coordinates. Its parameters are estimated by means of a least squares adjustment. It is assumed that the size of the remaining differences after the transformation is explicable by the uncertainty of the coordinates. This hypothesis is tested by means of a statistical test (Appendix C). As a result, the hypothesis is accepted, and consequently the identity of the places considered can be assumed. Figure 4 shows remaining deviations between the positions of GH 4.1 and the transformed positions of GH 4.6.

4.5 *Libya Interior* part 1: up to the *Saluentia* Capes

(3) River *Subos*, (4) River *Salathos*, (5) *Salathos*, (6) River *Chusarios*, (11) *Saluentia* Capes: for these places, identity with places in GH 4.1 is assumed according to Sect. 4.4 so that their localisations are adopted (see Table 2). On no. 11, also see below.

(27) *Sagapola* Mountains, middle: the (3) *Subos*/Oued Sebou has its source in these mountains (GH 4.6.8); the source of the Oued Sebou is in the **Middle Atlas**.

(94) Island of *Kerne*: Spaul (1958, p. 8.10): one of the Canary Islands. According to *Peripl.* 8, 10, *Kerne* has a circumference of 5 st ≈ 1 km and the journey time from the Pillars of Herakles (at the Strait of Gibraltar) to *Kerne* corresponds to that from *Karchedon* (*Carthage*) to the Pillars. It is often identified as **Herne** in the Bahia de Rio de Oro (e.g. Bunbury, 1879, p. 324, following Carl Müller), which roughly meets the information (the length of the sea route around Herne is ca. 2.5 km). Pliny states (NH 6.36) that according to Polybios *Kerne* is situated 8 st from the coast opposite the mountain *Atlas*. The former is in accordance with Herne, whereas the latter is not. Ptolemy makes the same mistake by positioning *Kerne* and the (M24) Bigger *Atlas* at similar latitudes (25°40′ and 26°30′).

(48) *Autolalai*: it is the northernmost town at the coast of *Libya Interior* south of Ptolemy's *Mauritania Tingitana*, which can be related to the Autololes according to its name. Ptolemy seems to follow the information that the Autololes are situated south of *Mauritania Tingitana* (cf. NH 5.1).

(95) Island of *Hera* or *Autolala*: Gosselin (1798–1813, p. 160): Fédal; Mannert (1825, p. 630): Madeira; Müller (1902): Mogador. The island is also called *Juno* or *Junonia*, and Ptolemy assigns a further (97) *Juno* to the Fortunate Islands. Ptolemy places *Hera* at the same latitude as the town of (48) *Autolalai*, i.e. as the Autololes. Ptolemy may refer to the Purple Islands (also Forbiger, 1844, p. 892) because Pliny, referring to Juba, places them opposite the Autololes (NH 6.37). Hence, *Hera* is **Mogador** as Ptolemy's (M69) *Erytheia* (see Sect. 4.3, no. 69). Furthermore, Ptolemy may use Sebosus' information that *Junonia* is situated 750 Rmi = 6000 Ist from *Gades* (NH 6.37; Cádiz) because this is consistent with Ptolemy's distance (Table 4; Müller, 1902, also identifies Ptolemy's island as Sebosus' *Junonia*). *Junonia* is one of the Canary Islands; see Sect. 6. Juba assigns *Junonia* to the Fortunate Islands, whereas Sebosus does not. Possibly, this brought Ptolemy to assume two islands, so that he assigned (95) *Hera* to the Purple Islands and (97) *Juno* to the Fortunate Islands.

The adjustment of a transformation of coordinates in Appendix C revealed a scaling factor of about 2 between the latitudes of GH 4.1 and 4.6 from the (3) *Subos* to the (11) *Saluentia* Capes. Since Ptolemy's coast runs almost southward, the factor $\frac{1}{2}$ can be applied to the distances of GH 4.6 as a correction. Additionally, the distortion of the distances of GH 4.1 described by a_1 and a_2 (Eqs. 13, 14) has to be considered. By means of their average of 0.621, the correction factor for Eq. (12) becomes

$$a_4 = 0.5 \cdot 0.621 = 0.311. \tag{16}$$

(7) *Gannaria* Capes: according to the name (Greek *akra*), Ptolemy refers to more than one cape, which is usually not taken into account (see e.g. Forbiger, 1844, p. 880; S & G). The most distinctive cape near $\bar{s}_{6,7}(a_4)$ is the **cape at Dar Bouazza**. South and north of it, there are a small peninsula and a small headland, which may have been referred to. Alternatively, the cape at Casablanca, 20 km northeast of Dar Bouazza, is included additionally.

(8) River *Ophiodes*: the first large river with respect to $\bar{s}_{7,8}(a_4)$ is the **Oued Oum er-Rbia** with acceptable s.

(10) River *Nuios*: no rivers exist between the (8) *Ophiodes*/Oued Oum er-Rbia and the (11) *Saluentia* Capes/Cap Cantin. The **Oualidia lagoon**, however, is situated consistently with $\bar{s}_8^{10}(a_4)$ and $\bar{s}_{10,11}(a_4)$, which may have been mistaken for a river.

(11) *Saluentia* Capes: the name suggests more than one cape. One of them corresponds to the (M14) *Helios* Mountain/**Cap Cantin** (see Sects. 4.2, 4.4, no. M14). Three kilometres south-south-east of it, there is a small headland with a height of 50 m, which is possibly referred to. Also, Cap de Safi 20 km south of Cap Cantin comes into consideration.

The localisation of the places reveals that there are two further places in GH 4.1 and GH 4.6, which refer to the same location or region:

- (M12) *Asana* and (L8) *Ophiodes*,
- (M13) *Diur* and (L10) *Nuios*.

The *Diur* and the *Nuios* refer to the Sidi Moussa and Oualidia lagoon, which constitute a chain of lagoons (Hughes et al., 1992, p. 66). The inclusion of these places in the statistical hypothesis test of the identity of places results in an acceptance of this hypothesis (Appendix C).

4.6 *Libya Interior* part 2: up to the *Stacheir*

(12) River *Massa*: Forbiger (1844, p. 882), S & G: Oued Massa. The name of the river suggests the **Oued Massa**. In accordance with Ptolemy's information, it is situated further south than the (11) *Saluentia* Capes/Cap Cantin and has its source in the (26) *Mandron* Mountains/Atlas (see no. 26 below). $\hat{s}_{11,12}$, however, is much too small. Probably, Ptolemy had no accurate information about this very long distance.

Owing to the large distance between nos. 11 and 12, these places may originate from different data sources. Therefore, the correction factor a_4 (Eq. 16) applied to the northern distances is not used in Eq. (12) for the following southern distances.

(14) River *Daras*: Forbiger (1844, p. 882): Río de Oro; S & G: Oued Draa. The *Daras* probably corresponds to the *Darat* mentioned by Pliny (e.g. Forbiger, 1844, p. 881), where crocodiles were found. There exist two short rivers ($s \doteq 175$ km, 200 km) in the vicinity of $\hat{s}_{12}^{14}$; the name, however, argues for the **Oued Draa** at $s \doteq 212$ km. Furthermore, crocodiles existed there; cf. de Smet (1998).

(30) *Kapha* Mountains, middle: S & G (p. 447, fn. 182): a part of the High Atlas. The (14) *Daras*/Oued Draa has its source in these mountains (GH 4.6.9). The confluences of the Draa rise in the middle part of the High Atlas; before it reaches lower areas, the Draa flows through a valley between the Anti-Atlas and Jbel Saghro. A part of these three mountains may correspond to the *Kapha* Mountains and may have been regarded as the location of the source of the *Daras*. Ptolemy's position far to the south is explicable by the southeasterly direction of the Draa from the coast, which may have been assumed for the entire course of the river. Φ of X fits better than that of Ω (10°).

(96–101) Fortunate Islands: Ptolemy's first and northernmost island (96) *Aprositos* has a somewhat smaller Φ than the (12) *Massa*/Oued Massa. This is met by the **Canary Islands**. Fuerteventura is visible from the mainland (Keyser, 1993) so that this island must have been known. On the individual islands, see Sect. 6.

(17) Cape *Arsinarion*: since this cape is situated in the latitudes of the Fortunate Islands/Canary Islands, it is **Cap Juby**.

(15) Large Port: the port may have been at a river mouth; in the vicinity of $\hat{s}_{14,15}$, however, there are only very short rivers. The **Khnifiss Lagoon** (Sebkha Tazra) is situated in compliance with $\hat{s}_{14,15}$, which may have been suitable for a port. Accordingly, Arlett (1836, p. 298) assumes that the Portuguese port Porto Consado shown on old maps of this region was at the lagoon. The name Large Port may have arisen from the extent of the lagoon.

(26) *Mandron* Mountains, middle: S & G: High Atlas. The five rivers nos. 4, 6, 8, 10 and 12 have their sources in the *Mandron* Mountains (GH 4.6.8). The locations of the actual sources are the following: (4) *Salathos*/Oued Bou Regreg: Middle Atlas; (6) *Chusarios*/Oued Mellah: at Khouribga, nearly halfway between the coast and Middle Atlas; (8) *Ophiodes*/Oued Oum er-Rbia: Middle Atlas; and (12) *Massa*/Oued Massa: Anti-Atlas. Thus, it can be assumed that the *Mandron* Mountains are the entire **mountain range of the Middle, High and Anti-Atlas**, whose middle is in the High Atlas. A part of the *Mandron* Mountains is already given in GH 4.1 by the (M24) Bigger *Atlas*. The (10) *Nuius* refers to the Oualidia lagoon; the source of this supposed river was assumed to be in easterly mountains, i.e. in the *Mandron* Mountains.

(28) *Rhussadion* Mountains, middle: Mannert (1825, p. 525): Cap Blanc; S & G (p. 447, fn. 180): possibly near Cape Ghir. The source of the (19) *Stacheir*/Wad As Saguia al Hamra is in the *Rhussadion* Mountains (GH 4.6.8). This river has tributaries in the **highland southwest of Jebel Ouarkziz**.

(18) Cape *Rusaddir*: according to Ptolemy, this cape is situated west of the (28) *Rhussadion* Mountains/highland southwest of Jebel Ouarkziz. This is in agreement with **Ras Afkir Oum M'Bark**. $\hat{s}_{17,18}$, however, does not support this identification; $\hat{s}$ of Ω fits better than that of X (X: no. 17 $\Phi = 13°$).

(19) River *Stacheir*: according to X, the river mouth is east of (18) Cape *Rusaddir*/Ras Afkir Oum M'Bark, which is not given by the direction of the coast. According to Ω, the river is further south than no. 18 so that it may be the **Wad As Saguia al Hamra**. It is the largest and longest watercourse in Western Sahara (Hughes et al., 1992, p. 90). This identification, however, is not substantiated by $\hat{s}_{18,19}$.

4.7 *Libya Interior* part 3: up to the *Ochema Theon*

(29) *Ochema Theon* Mountain: according to *Peripl.* 16, 17, flames and torrents of fire were observed on this mountain so that it is probably a volcano. Burton (1862) identifies it as Mount Cameroon, probably for the first time, which was the only volcano then active in the area under investigation (cf. Hennig, 1944, 86–95). Forbiger (1844, p. 880, fn. 18) assumes that the appearance of fire was not caused by a volcano but by people, which, however, does not explain the mentioned torrents of fire. Mount Cameroon at 4°13′ latitude nearly fits $\Phi = 5°$ of the *Ochema Theon*. The source of the (24) *Masitholos*/Niger (see no. 24 below) is situated at the *Ochema Theon* (GH 4.6.9). The Niger has a tributary, the Benue, which has its sources in the Western High Plateau north-east of Mount Cameroon. Hence, Ptolemy refers to **Mount Cameroon and the Western High Plateau**, which explains why he does not place the *Ochema Theon* directly at the coast.

Ptolemy's distance from the (19) *Stacheir*/Wad As Saguia al Hamra to the (29) *Ochema Theon*/Mount Cameroon over nos. 19–24 and 29 is a few thousands kilometres too short. Hence, some or all Ptolemaic distances south of no. 19 are strongly shortened. The southern regions were surely less navigated so that only little and fragmentary information was available. A main reason for the shortening, however, is Ptolemy's repetition of a part of *Mauritania Tingitana* in GH 4.6, which diminished the available space. Additionally, the coast is shortened through its wrong direction from north-west to south-east. This direction corresponds to the ancient idea of the western side of Africa (e.g. G 2.5.15, Mela 1.20; cf. Berger, 1903, p. 400; Romer, 1998, p. 39, fn. 19), which Ptolemy probably makes use of. Consequently, the following localisations are mostly not based on distances.

(22) River *Nias*: Mannert (1825, p. 526), Forbiger (1844, p. 882), S & G: Senegal. Following this suggestion, the *Nias* is assumed to be the **Senegal**.

Like the *Nias*, the *Chretes* in *Peripl.* 9 and the *Bambotus* in NH 5.1 are also equated with the Senegal (e.g. S & G, p. 447, fn. 177; Winkler and König, 1993, p. 352). The equating of the *Chretes* with the Senegal is questionable. The text of *Peripl.* 9,10 reads: "Sailing from there [*Kerne*/Herne], we crossed a river called Chretes, and reached a bay, which contained three islands [...] After a day's sail from here, we arrived at the end of the bay, which was overhung by some very great mountains. [...] Leaving from there, we arrived at another large, broad river teeming with crocodiles and hippopotamuses." The passage suggests that the three islands are not more than a few days' journey distant from *Kerne*. This is only met by the three islands at Cap d'Arguin, which are about 450 km from *Kerne*, or by Tidra and its surrounding islands. The *Chretes* is further north and cannot be the Senegal. The mentioned bay is probably the section between Cap d'Arguin (16°32′ W/20°33′ N) and Cap Tafarit (16°16′ W/20°08′ N), and the islands are therefore at Cap d'Arguin. The actual distance of 55 km between both capes is in accordance with the given journey time if it is rounded up to 1 D. The *Chretes* may refer to the Bay of Lévrier (16°54′ W/20°53′ N) and the salt marshes at its shores, which may have been taken for a river mouth. The mentioned mountains possibly correspond to Cap Tafarit, which has an extent of about 1 km and, in contrast to its vicinity at ground level, a height of about 40 m. The mentioned second river may be the Senegal (also Forbiger, 1844, p. 882; Lendering, 1998–2014), where there are crocodiles today (e.g. Hughes et al., 1992, p. 48).

(21) Cape *Katharon*: the most significant cape between the (19) *Stacheir*/Wad As Saguia al Hamra and the (22) *Nias*/Senegal is **Cap Blanc**.

(23) *Hesperu Keras* ("Horn of the West"): Mannert (1825, p. 525), Forbiger (1844, p. 881), Bunbury (1879, p. 325, fn. 6): Cap-Vert. A place of the same name is to be found in *Peripl.* 14 and NH 5.1, 6.35, 36. According to the *Periplus*, it is a bay, which is identified as the Bight of Benin (e.g. Winkler and König, 1993, p. 121). Later authors, including Pliny and Ptolemy, transformed it into a cape (cf. Bunbury, 1879). According to *Peripl.* 14, 16, an island in the bay is a 4-day journey from the (29) *Ochema Theon*/Mount Cameroon. This journey time is also specified in NH 6.35 for the distance *Hesperu Keras*–*Ochema Theon* so that Pliny and probably also Ptolemy actually refer to the bay. The journey time is consistent with Ptolemy's $\hat{s}_{23}^{29}$ (over nos. 23–25 and 29; see Sect. 3.2.2, Table 4) and corresponds to about 740/630 km. The **Bight of Benin** is situated at this distance.

(24) River *Masitholos*: Mannert (1825, p. 532), Forbiger (1844, p. 882): Gambia; Lacroix (1998, p. 264): Volta; S & G: Oued Massa. The river has to be between the (23) *Hesperu Keras*/Bight of Benin and the (29) *Ochema Theon*/Mount Cameroon. The **Niger** Delta extends over this area. The source of the *Masitholos* is in the (29) *Ochema Theon*/Western High Plateau (GH 4.6.9), which is met by the river **Benue**, a tributary of the Niger.

(2) Hesperian Gulf: Lacroix (1998, p. 263): Gulf of Guinea. The Hesperian Gulf lies south of the (24) *Masitholos*/Niger Delta. It may be the **Gulf of Guinea**. The places from the (19) *Stacheir*/Wad As Saguia al Hamra southwards, however, are also situated at the gulf (GH 4.6.7). This probably follows from a wrong idea of the direction of the coast; cf. Fig. 2. Thus, in the strict sense, Ptolemy's Hesperian Gulf refers to the Atlantic Ocean west and south of Western Africa.

5 Errors of Ptolemy's data

Based on the localisations of the Ptolemaic places described in Sect. 4, the errors of the Ptolemaic distances (Sect. 5.1) and coordinates (Sect. 5.2) are investigated.

5.1 Distances

In order to illustrate the errors of the Ptolemaic distances $\hat{s}$, the direct conversion of Eq. (10) is used for their derivation from the Ptolemaic coordinates. If identifications are missing between two places, the sum Eq. (11) is used; no. M9 is, however, excluded from it. Tables 7 and 8 give the deviation

$$e = \hat{s} - s \tag{17}$$

between $\hat{s}$ and the actual distance s along the coast. The strongly distorted distances south of the (L12) *Stacheir* are not considered; on their errors, see Sect. 4.7.

A few $\hat{s}$ have small deviations e of some kilometres. They are explicable by rounding errors of distances and coordinates, which may have caused distance errors of about 20 km (GH 4.1) or 30 km (GH 4.6); cf. Sect. 3.2.1. Several deviations, however, are significantly larger.

If the $\hat{s}$ are based on the Italian stade (Eq. 5) in Eq. (10), then the majority of them are larger than the corresponding s. If the Egyptian stade (Eq. 6) is used, then the cases $\hat{s} > s$ and $\hat{s} < s$ occur almost equally frequently. There are coastal stretches where contiguous distances $\hat{s}$ are too large or too small, respectively ($\hat{s} > s$: nos. M4–M10, M17–M22, L15–L19, partly L3–L11; $\hat{s} < s$: M11–M14, L11–L15). If Ptolemy diminished a distance due to the curvilinearity of the route and anomalies of the journey (cf. Sect. 2.2), then the Ptolemaic distance $\hat{s}$ should be smaller than the curvilinear actual

Table 7. Deviations $e = \hat{s} - s$ between the Ptolemaic distances $\hat{s}$ Eq. (10) and the actual distances s in GH 4.1 *Mauritania Tingitana*. S.: source of the Ptolemaic position; Ist, Est: based on the Italian/Egyptian stade.

From/To	S.	$\hat{s}$ (km)		s	e (km)	
		Ist	Est	(km)	Ist	Est
1 C. *Kotes*	–	–	–	–	–	–
2 *Zilia*	X	54	46	29	25	17
3 *Lix*	Ω	46	39	40	6	-1
4 *Subur*	Ω	54	46	114	−60	−68
6 *Sala* (river)	X	49	42	29	20	13
7 *Sala*	X	33	28	3	30	25
8 *Duos*	X	41	35	33	8	2
10 *Kusa*	X	93	79	33	60	46
11 *Rusibis*	Ω	23	20	23	0	−3
12 *Asana*	Ω	53	45	75	−22	−30
13 *Diur*	Ω	41	34	60	−19	−26
14 *Helios* Mtn.	Ω	60	51	69	−9	−18
15 *Mysokaras*	Ω	60	51	31	29	20
16 *Phthuth*	Ω	34	29	32	2	−3
17 C. of Herakles	Ω	46	39	49	−3	−10
18 *Tamusiga*	Ω	41	35	23	18	12
19 C. *Usadion*	Ω	74	63	15	59	48
21 *Una*	Ω	93	79	36	57	43
22 *Agna*	Ω	82	70	43	39	27
23 *Sala*	Ω	54	46	53	1	−7
24 Bigger *Atlas*	Ω	82	70	79	3	−9

distance s. Hence, a general shortening of distances is not indicated by the data.

Mostly, large deviations e can be explained satisfactorily by the types of errors given in Sect. 3.2.1. Two examples shall be given.

The distance (M4) *Subur*–(M7) *Sala* may have been altered by error 2a. The distance is also given by Pliny and was probably specified as 1 D or $\frac{1}{2}$ DN; cf. Sect. 3.2.2 and Table 4. The actual distance $s = 29\,\text{km} + 3\,\text{km} = 32\,\text{km}$ and a northward voyage with speed v_{sn} (Eq. 8) lead to a journey time of 0.43/0.51 days (Ist/Est), which is in accordance with $\frac{1}{2}$ DN. When Ptolemy's derivation of the distance was based on the rounded journey time of 0.5 days and on the larger speed $v_{\varnothing}$ (Eq. 3; see Sect. 3.2.2), then his distance became too large. The erroneous partial distances (4) *Subur*–(6) River *Sala*–(7) *Sala* may originate from a halving of 1 D. $\frac{1}{2}$ D corresponds to 250 st (Eq. 4 according to Sect. 3.2.2). Consistent with this, Ptolemy's distances amount to $\hat{s}_4^6 = 263$ st and $\hat{s}_{6,7} = 266$ or 180 st (Ω/X).

The cause of the overly large distance (L11) *Saluentia* Capes–(L12) *Massa* may be insufficient information (Sect. 4.6, no. 12). Alternatively, it may have been altered by errors 2b and 3b. The actual s and a southward voyage with v_{ns} (Eq. 7) yield a journey time of 1.27/1.49 days (Ist/Est). Possibly, this was specified as $1\frac{1}{2}$ DN and confused with $1\frac{1}{2}$ D. The determination of a distance by means of the

Table 8. Deviations $e = \hat{s} - s$ between the Ptolemaic distances $\hat{s}$ Eq. (10) and the actual distances s in GH 4.6 *Libya Interior*. S.: source of the Ptolemaic position; Ist, Est: based on the Italian/Egyptian stade.

From/To	S.	$\hat{s}$ (km)		s	e (km)	
		Ist	Est	(km)	Ist	Est
3 *Subos*	–	–	–	–	–	–
4 *Salathos* (river)	Ω	279	237	29	250	208
5 *Salathos*	Ω	29	24	3	26	21
6 *Chusarios*	Ω	42	36	64	−22	−28
7 *Gannaria* C.	Ω	116	99	43	73	56
8 *Ophiodes*	Ω	63	54	55	8	−1
10 *Nuios*	Ω	235	199	96	139	103
11 *Saluentia* C.	Ω	89	76	32	57	44
12 *Massa*	Ω	128	109	305	−177	−196
14 *Daras*	Ω	149	127	212	−63	−85
15 Large Port	Ω	93	79	132	−39	−53
17 C. *Arsinarion*	Ω	347	295	68	279	227
18 C. *Rusaddir*	Ω	65	55	39	26	16
19 *Stacheir*	Ω	102	87	61	41	26

smaller speed $v_{\varnothing}$ results in 750 st, which is in accordance with Ptolemy's $\hat{s} = 692$ st.

The following errors of distances are possibly due to an intentional alteration by Ptolemy. The distance (L3) *Subos*–(L4) *Salathos* may have been enlarged by Ptolemy in order to integrate (L95) *Hera*/*Autolala* and (L48) *Autolalai* between the *Subos* and the *Salathos* (see Sect. 4.5, no. 48). The oversized distance (L15) Large Port–(L17) Cape *Arsinarion* is probably caused by the preset longitudinal distance of 60°30′ between the Fortunate Islands and *Alexandria* (see Sect. 6). As a result, the distance of these islands from the African coast became too large. Ptolemy possibly shifted Cape *Arsinarion* to the west in order to compensate for this error and emphasise its position opposite the Fortunate Islands.

The adjustment of a transformation between the coordinates of GH 4.1 and 4.6 in the region from the Oued Sebou to Cap Cantin (Appendix C) reveals that the distances between the places concerned of GH 4.6 are significantly larger than those of GH 4.1. This is explicable by Ptolemy's enlargement of the distance L3–L4 (see above), by the increase in the distances L6–L7, L8–L10–L11 (e.g. error 2a) and by the decrease in the distances M11–M12–M13–M14.

5.2 Coordinates

Ptolemy's African western coast runs wrongly almost from north to south (Figs. 1 and 2). Longitudinal distances between places are often grossly erroneous and coastal stretches are wrongly oriented. The reason is surely fragmentary and erroneous information. For example, the *Periplus* deals with more than 30 places, but only five directions between them are indicated, two of them southward and two eastward. Such insufficient and erroneous information may have easily caused a wrong idea of the coastal direction. There are, however, some coastal stretches with approximately correct directions, for example (M7) *Sala*–(M8) *Duos* and (M13) *Diur*–(M14) *Helios* Mountain.

Ptolemy's latitudes of the northernmost places and of the southernmost location (L29) *Ochema Theon* are nearly correct. The northern latitudes in *Libya Interior* are shifted significantly southwards owing to Ptolemy's arrangement of *Libya Interior* south of *Mauritania Tingitana* and his repetition of a part of *Mauritania Tingitana* in *Libya Interior* (Sect. 4.4).

Ptolemy's latitudinal difference $\Delta\Phi$ between (M1) Cape *Kotes* and the (M24) Bigger *Atlas* amounts to $9\frac{1}{2}^{\circ}$, but the actual difference is about 6°. $\Delta\Phi$ between the (L3) *Subos* and (L17) Cape *Arsinarion* is 13°; the actual difference, however, is about 6°. Thus, the latitudinal extent north of (L17) Cape *Arsinarion* is too large, which is explicable by the following. First, Ptolemy used 500 st per degree, which is too low and yields too large latitudinal differences (also Mannert, 1825, p. 480). Second, the wrong southward direction of the coast takes coastal places further south. Third, the lengths of some coastal stretches are enlarged.

$\Delta\Phi$ of (L17) Cape *Arsinarion* and the (L29) *Ochema Theon* is 7°, but the actual latitudinal difference amounts to about 21°. This shortening is explicable by the southward shift of the places in *Libya Interior* and by a measured latitude in the region of the *Ochema Theon*, which constituted a southern limit.

6 The Fortunate Islands

The consideration of the repetition of a part of GH 4.1 *Mauritania Tingitana* in GH 4.6 *Libya Interior* reveals that Ptolemy's Fortunate Islands correspond to some of the Canary Islands (Sect. 4.6). This is confirmed by the information of other ancient authors. In the following, this information is considered (Sect. 6.1) and the single islands of the Fortunate Islands are identified (Sect. 6.2).

6.1 Further ancient information

In several partly mythological ancient texts, islands west of Africa are mentioned (see e.g. Keyser, 1993). In Sallust's *Historiae* (frr. 1.100–1 M; e.g. Keyser, 1993) and in Plutarch's *Lives* (Sertorius 8.2; e.g. Keyser, 1993) two Fortunate Islands occur. They are taken for some of the Canary Islands (e.g. Gosselin, 1798–1813, p. 147), or they are identified as Madeira and Porto Santo, and it is assumed that their name was transferred to the Canary Islands later (Müller, 1902; Fischer, 1910; Keyser, 1993). According to Sallust and Plutarch, the two Fortunate Islands are 10 000 st distant from *Gades*/Cádiz and *Libya*, respectively. The latter possibly refers to the Strait of Gibraltar (similarly e.g. Mannert, 1825, p. 623; Keyser, 1993). Ptolemy probably made use of the former distance, since his distance *Gades*–Fortunate Islands (*Aprositos*) amounts to $\hat{s} = 10\,252$ st (Eq. 9, steps 1 and 2).

According to Pliny's description of the Fortunate Islands (NH 6.37), they are beyond the Purple Islands/Mogador. In his description he refers to Sebosus and Juba; their information on the islands is compiled in Table 9 (based on Brodersen (1996); Bostock and Riley (1855) have some deviations).

Sebosus distinguished between the three islands of *Junonia*, *Pluvialia* and *Capraria* and the two Fortunate Islands of *Invallis* and *Planasia*. *Junonia* is 750 Rmi = 6 DN distant from *Gades*/Cádiz (Table 4). This distance is probably based on the journey time of a sea route from Cádiz along the African coast to Cap Sim and from there southwestwards to the Canary Islands. Its length is about 1150 km and corresponds to 5/6 DN (Ist/Est; reversion of Eq. 9, steps 5, 4, $v_4 = v_{\mathrm{ns}}$ Eq. 7). *Pluvialia* and *Capraria* are 750 Rmi from *Junonia* in a westerly direction; the two Fortunate Islands are 250 Rmi from *Pluvialia* and *Capraria* in a southwesterly direction. If these distances refer to western islands of the Canary Islands, then they are much too large.

Juba mentioned six Fortunate Islands, of which five islands probably correspond to Sebosus' islands; see Table 9. *Ninguaria* received its name from perpetual snow; it may be Sebosus' *Invallis* (Latin vaulty), whose name is possibly a corruption of *nivalis* (Latin snowy; Fischer, 1910 according to J. Partsch and Carl Müller; Brodersen, 1996, p. 248). *Canaria* may be Sebosus' *Planasia* (see below). *Ombrios* is the Greek name of Sebosus' *Pluvialia* (Müller, 1902). According to Juba, the Fortunate Islands are "... at a distance from the Purple Islands of six hundred and twenty-five miles [5 DN], the sailing being made for two hundred and fifty miles [2 DN] due west, and then three hundred and seventy-five [3 DN] towards the east." Müller (1902) points out that the distances correspond to 5000, 2000 and 3000 st and that they are rough estimates based on 1 DN = 1000 st (i.e. Eq. 4). The reported directions are contradictory. A westward seafaring from the Purple Islands/Mogador is diverted to the south due to the Canary Current and the trade winds. Possibly a southwestern journey over the sea to the Canary Islands (Alegranza) is meant, which has a length of about 425 km. This corresponds to 2 DN (calculation as above) in agreement with the distance of Juba's first, westward route. The second sea route of 375 Rmi may be a journey to further islands of the Canaries and along their coasts. Its eastward direction is wrong but explicable by a return journey from western to eastern islands.

According to the distance data of Sebosus and Juba, the Fortunate Islands may be some of the Canary Islands. This is substantiated by Mela, who states: "On the sandy part is Mt. Atlas [...] Opposite the sandy part, the Fortunate Isles abound ..." (3.101, 102). Accordingly, the Fortunate Islands are situated in the latitudes of the Sahara and the Atlas mountains. This is met by the Canary Islands.

Table 9. Information about the Fortunate Islands from Sebosus and Juba (NH 6.37, translation by Brodersen, 1996).

Ptolemy	Sebosus	Juba
–	–	All islands: 625 Rmi southwest of the Purple Islands; high in fruits and birds of every kind
96 *Aprositos*	–	–
97 *Juno*	1. *Junonia*: 750 Rmi from *Gades*	2. *Junonia*: a small temple
98 *Pluvialia*	2. *Pluvialia*: 750 Rmi west of no. 1; the only freshwater is rainwater.	1. *Ombrios*: no buildings; among the mountains a swamp, trees similar to the giant fennel, from which a bitter and a drinkable sap are extracted
99 *Capraria*	3. *Capraria*: 750 Rmi west of no. 1	4. *Capraria*: many huge lizards
100 *Canaria*	5. *Planasia* (Fortunate Islands): 250 Rmi southwest of nos. 2, 3	6. *Canaria*: name owing to the many big dogs; traces of buildings; many date palms, pine nuts, much honey; in the rivers papyrus, fish
101 *Centuria* (*Pintuaria*)	4. *Invallis* (Fortunate Islands): 250 Rmi southwest of nos. 2, 3; name from its undulating surface; circumference of 300 Rmi; trees up to 140 feet	5. *Ninguaria*: within sight of no. 4; perpetual snow; fog
–	–	3. smaller *Junonia*: smaller than no. 2; in the vicinity of no. 2

6.2 Identification of the islands

The identification of the islands is dealt with by, e.g. Gosselin (1798–1813, 146–159), Buch (1819), Müller (1902); Krüss (1976) gives further works in this regard and discusses the etymology of the islands.

Five of Ptolemy's six Fortunate Islands probably correspond to Sebosus' and Juba's islands; see Table 9. Four islands have identical or similar names; (101) *Centuria* (*Pintuaria*) may be *Ninguaria* (Fischer (1910) according to a conjecture by Carl Müller and Curt Müller). It was shown in Sect. 3.2.2 that Sebosus' distances are consistent with Ptolemy's distances. Ptolemy's wrong arrangement of the islands from north to south is contradictory to Sebosus' directions, but a rotation of Ptolemy's positions brings them nearly in accordance with Sebosus' information.

(100) *Canaria*: the name has been preserved in **Gran Canaria** to date (likewise e.g. Gosselin, 1798–1813, p. 155). Date palms and a species of pine (Juba) are characteristic of Gran Canaria (Müller, 1902).

The Canary Islands consist of seven large and further smaller islands. An assignment of the Fortunate Islands to the large islands leads to contradictions. If *Canaria* is Gran Canaria, then there are no three large islands in the east which could be the counterparts for the eastern islands *Junonia*, *Pluvialia* and *Capraria*. Buch (1819), for example, identifies *Junonia* with Fuerteventura and *Pluvialia* with Lanzarote, which contradicts the position of *Pluvialia* west of *Junonia* (Sebosus). Ptolemy and Juba mention only six islands so that (at least) one of the large Canary Islands does not occur. Therefore it is also possible that more than one large island is missing, and the small islands from Graciosa to Alegranza come into consideration. Through their inclusion, a solution can be found which is consistent with Sebosus' and Ptolemy's arrangement. The small islands have a largest extent of 2 to 9 km. Ptolemy lists islands with comparable sizes in other regions (e.g. *Planasia*/Pianosa in GH 3.1, *Erikodes*/Alicudi in GH 3.4; see Marx and Kleineberg, 2012, 26, 47).

(101) *Centuria* (*Invallis*, *Ninguaria*): owing to the perpetual snow (Juba), it may be **Tenerife** with the snow-covered Mount Teide (e.g. Buch, 1819; Müller, 1902). The fog (Juba) may refer to clouds being present around Mount Teide for a large part of the year (Müller, 1902). Sebosus gives a circumference of 300 Rmi. The islands with the largest circumference (sea route) are Tenerife and Lanzarote with about 250 km $\approx$ 170 Rmi. The circumference was possibly enlarged to 300 Rmi in order to take into account the jointed coast.

(98) *Pluvialia* (*Ombrios*): the lack of water (Sebosus) applies to **Lanzarote** (also Müller, 1902); see e.g. Chisholm (1911, vol. 16, p. 118). Buch (1819) and Müller (1902) mention a swamp and accumulation of water, respectively, on Lanzarote, which Juba's lake may refer to. According to Buch (1819), the trees from which the sap is extracted may be the *Euphorbia canariensis* and the *Euphorbia balsamifera*.

(99) *Capraria*: Ptolemy places the island opposite to (17) Cape *Arsinarion*/Cape Juby, which is met by **Fuerteventura**. According to Müller (1902), the name indicating a multitude of goats applies particularly to Fuerteventura. (101) *Centuria*/Tenerife is within sight of *Capraria* (Juba), which holds true for Fuerteventura (Müller, 1902).

(96) *Apròsitos*: this island is the northernmost island and may therefore be **Alegranza**, which is the northernmost of the Canary Islands. The name *Aprositos* means inaccessible, which applies to Alegranza; Arlett (1836) reports that there exists only one landing place on the southern side.

(97) *Juno* (*Junonia*): according to Sebosus, this island is the nearest to *Gades*, so that it is further north than *Pluvialia*/Lanzarote. Ptolemy positions it between *Pluvialia*/Lanzarote and *Aprositos*/Alegranza. Both conditions apply to **Graciosa** (also identified by Müller, 1902).

Smaller *Junonia*: it was suggested that Juba's smaller *Junonia* (Juba) corresponds to (96) *Aprositos* (see Fischer, 1910), which is not followed here. Pliny's text suggests that this *Junonia* is near to *Junonia*/Graciosa so that it is probably **Isla de Montaña Clara**.

Planasia: Sebosus' *Planasia* may be Ptolemy's and Juba's *Canaria*/Gran Canaria (also suggested by Müller (1902), who assumes a corruption of the name). Sebosus mentioned Fuerteventura (*Capraria*) and Tenerife (*Invallis*) so that the interjacent Gran Canaria should have been known to him. Sebosus gives the same relative position for *Planasia* and for *Invallis*/Tenerife with respect to *Junonia* so that *Planasia* and *Invallis* must be near to each other; the island nearest to Tenerife in the east is Gran Canaria. The name *Planasia* means plane (from Latin *planus*) and is probably chosen in view of the name of the island of *Invallis*/Tenerife; consistent with this, Gran Canaria is less high than Tenerife (likewise Gosselin, 1798–1813, p. 151).

According to Juba, the Fortunate Islands are rich in fruits and birds of every kind, and Mela states: "... the Fortunate Isles abound in spontaneously generated plants; and with various ones always producing new fruit in rapid succession, the islands nourish people who want for nothing, whose islands are more blissfully productive than others are" (3.102). Hence, the good conditions for the vegetation growth gave the Fortunate Islands their name. This feature applies to Gran Canaria and Tenerife but not to Fuerteventura and Lanzarote, where the vegetation is sparse (cf. e.g. Chisholm, 1911, 174 pp.). It seems that this was taken into account by some ancient authors, since Sebosus did not assign the islands east of Gran Canaria to the Fortunate Islands.

7 Ptolemy's prime meridian

In his *Mathematike Syntaxis* and in GH Book 8, Ptolemy uses a prime meridian which passes trough *Alexandria* (see Sect. 1). For the catalogue of locations in GH books 2–7, he introduced a different prime meridian. The location of this prime meridian is considered in the following (Sect. 7.1), and the origin of its position is investigated (Sect. 7.2).

7.1 Location of the prime meridian

The geographic coordinate system which underlies Ptolemy's position data in GH Books 2–7 is described in GH 1.19.2. The longitude is counted with respect to a westernmost meridian which constitutes the western end of the known world (1.19.2, 1.22.4, 1.24.9). This western end or prime meridian, respectively, is at the Fortunate Islands (1.11.1, 7.5.15, 8.27.12). In the catalogue of locations, the Fortunate Islands have $\Lambda = 1°$ in Ω (4.6.34); deviating from this, the four islands *Aprositos*, *Pluvialia*, *Capraria* and *Centuria* (*Pintuaria*) have $\Lambda = 0°$ in *X*. The value of 1° is a substitute for 0° due to the lack of a common sign for the value zero (S & G, p. 455, fn. 202). The longitude of *Alexandria* is $\Lambda = 60°30'$ (4.5.9). Furthermore, the longitudinal differences between *Alexandria* and the Fortunate Islands of

$$\Delta\Lambda_{\mathrm{F,A}} = 60°30' \tag{18}$$

as well as of 4 h are given (1 h $\hat{=}$ 15°; 7.5.14, 8.15.10, indirectly 8.27.12). The former yields 4 h 02 min; the latter corresponds to 60° and is a rounded value.

The localisation of Ptolemy's Fortunate Islands (Sect. 6) revealed that the westernmost of these islands is Tenerife. Therefore it can be regarded as the location of Ptolemy's prime meridian, which constitutes the western end of the known world.

For the definition of the new prime meridian at the Fortunate Islands, the longitudinal extent of the known world in the west had to be known. This, however, does not mean that Ptolemy had to determine the longitudinal distances of a multitude of places between the meridians of *Alexandria* and the Fortunate Island, since he adopted only a few primary distances from Marinos.

7.2 Origin of the position of the prime meridian

In ancient geography, the longitudinal extent of the known world was specified by means of the arc lengths r on the parallel through *Rhodos*. A comparison of such or comparable distances given by ancient authors is carried out by e.g. Blair (1784, 118–128) and Berggren and Jones (2000, 153–154). From the *Rhodos* parallel it was assumed that it runs through the Pillars of Herakles at the Strait of Gibraltar. The origin of Ptolemy's longitudinal difference $\Delta\Lambda_{\mathrm{F,A}}$ (Eq. 18) becomes evident by a consideration of the distances r_{P} with respect to the Pillars of Herakles. In addition, arc lengths r_{M} with respect to the Strait of Messina are considered. Table 10 compares some arc lengths r_{P} and r_{M} from the Atlantic to *Issos* (Dörtyol at the Gulf of Iskenderun, S & G) originating from Eratosthenes (G 1.4.5), Agrippa (NH 6.38), Polybios (NH 6.38), Strabo (G 2.1.40, 2.4.3), Marinos (GH 1.12.11) and Ptolemy (GH 2.4.6, 2.5.3, 3.4.9, 4.3.7, 5.2.34, 5.8.4); partly they are sums of further distances. Polybios' (ca. 200–118 BC) and Agrippa's distances are given in Rmi by Pliny; they are converted by 1 Rmi = 8 st into st. Ptolemy assumes 400 st per 1° for the *Rhodos* parallel (GH 1.11.2); his and Marinos' longitudinal differences are converted by means of this value. In the case of Ptolemy, Mountain *Calpe* (Rock of Gibraltar, $\Lambda = 7°30'$) is used for the Pillars, *Messene*

Table 10. Arc lengths on the parallel of *Rhodos* from different ancient sources expressed in st. r_P, r_M: arc length with respect to the meridian of the Pillars of Herakles/the Strait of Messina.

Location	Eratosthenes	Polybios	Agrippa	Strabo	Marinos	Ptolemy
r_P						
Western end of the known world	−5000	–	–	–	–	–
Western end of Europe	−3000	–	–	–	–	–
Fortunate Islands	–	–	–	–	−3000	−3000
Sacred Cape	–	–	–	$\doteq$ −3000	−2000	−2000
Pillars of Herakles	0	0	0	0	0	0
Rhodos	$\doteq$ 21 500	14 500	–	$\doteq$ 20 500	20 300	20 200 to 20 467
Alexandria	$\doteq$ 21 500	–	–	$\doteq$ 20 500	–	21 200
(Gulf of) *Issos*	–	$\doteq$ 19 520	27 520	$\doteq$ 25 500	24 800	24 733
r_M						
Karchedon	0	–	–	0	–	−1667
Strait of Messina	0	0	0	0	0	0
Rhodos	$\doteq$ 13 500	$\doteq$ 4500	–	$\doteq$ 8500	–	7400 to 7667
Alexandria	$\doteq$13 500	–	10 800	$\doteq$ 8500	–	8400

(Messina, $\Lambda = 39°30'$, S & G) for the Strait of Messina and the westernmost and easternmost places on *Rhodos* ($\Lambda = 58°$ and $58°40'$) for *Rhodos*.

Polybios' r_P to the Gulf of *Issos* and r_M to *Rhodos* apply only approximately because Polybios refers to *Seleucia Pieria* (near the Gulf of *Issos*) and to Sicily, respectively. His distances are not consistent with the other ancient sources and are not further considered.

7.2.1 Marinos

In his discussion of the geographical works of Marinos, Ptolemy reports longitudinal distances $\Delta\Lambda$ from the Fortunate Islands to the *Euphrat*. Ptolemy complies closely with Marinos' distances (Berggren and Jones, 2000); the distances of Table 10 are in accordance. Apart from islands, the westernmost place of the entire *Geography* is the Sacred Cape (Cabo de San Vicente, S & G) in *Hispania Lusitania*. Ptolemy's $\Delta\Lambda = 2°30'$ from the Fortunate Islands to the Sacred Cape was already specified by Marinos; this also applies to Ptolemy's $\Delta\Lambda = 7°30'$ from the Fortunate Islands to the Pillars of Herakles. Marinos' positioning of *Alexandria* is unknown, but it is conceivable that also Ptolemy's $\Delta\Lambda_{\mathrm{F,A}}$ resulted from Marinos' data.

7.2.2 Eratosthenes

Strabo does not state explicitly that Eratosthenes' distances apply to the *Rhodos* parallel, but this is usually assumed (e.g. Thomson, 1948, 164–165). Eratosthenes' distances do not refer to *Alexandria* but to the Nile at *Kanobos* (Abukir, S & G); owing to the closeness of both locations, however, the distances also apply approximately to the *Alexandria*–*Rhodos* meridian. Eratosthenes arranged a distance of $r_\mathrm{P} = -3000$ st for the bulge of Europe. Furthermore, he added a second distance of 2000 st probably for promontories and islands and in order to obtain a longitudinal extent of the known world which is more than double the latitudinal extent (cf. G 1.4.5). (Eratosthenes applied both distances also to the eastern end of the known world.)

For the later geographers Marinos and Ptolemy, the region of the 2000 st was obviously meaningless. First, the longitudinal and latitudinal extent was determined by considerations other than those mentioned by Strabo; see GH 1.7, 1.11. Second, Ptolemy's and probably also Marinos' westernmost place of Europe is the Sacred Cape, which is only $\Delta\Lambda = 5°$ from the Pillars (*Calpe*), i.e. $r_\mathrm{P} = -2000$ st, so that Eratosthenes' first distance of -3000 st is not reached. This distance, however, was suitable for islands in the Atlantic whose location was not accurately known and which were assumed to be the westernmost ones. Marinos probably used Eratosthenes' distance and located the Fortunate Islands 3000 st ($\Delta\Lambda = 7°30'$) from the Pillars. This corresponds to Ptolemy's longitude of Mountain *Calpe*. Ptolemy's $\Delta\Lambda = 53°$ between Mountain *Calpe* and *Alexandria* yields $r_\mathrm{P} = 21\,200$ st and is probably based on Eratosthenes' $r_\mathrm{P} = 21\,500$ st, which corresponds to $\Delta\Lambda = 53°45'$. This value may have been rounded to 53°, possibly because Eratosthenes' distance refers to the Nile east of *Alexandria*. Consequently, Ptolemy's $\Delta\Lambda_{\mathrm{F,A}}$ or $r = 24\,200$ st, respectively, from the Fortunate Islands at the western end of the known world to *Alexandria* is based on Eratosthenes' $r = 24\,500$ st from an assumed western end of Europe to *Alexandria*.

7.2.3 Strabo

Strabo reports further distances from sources not indicated. His values given in Table 10 apply only approximately, since Strabo specifies some distances imprecisely. His r_P to *Rhodos* corresponds approximately to Marinos' and Ptolemy's distances. Therefore, Marinos' and Strabo's distances may have been derived from an identical source. Strabo states: "… Eratosthenes says […] that the distance from Alexandria to Carthage [*Karchedon*] is more than thirteen thousand stadia, though it is not more than nine thousand – if Caria [region in the southwest of Turkey] and Rhodes lies, as Eratosthenes says, on the same meridian as Alexandria, and the Strait of Sicily [Strait of Messina] on the same meridian as Carthage. In fact, all agree that the voyage from Caria to the Strait of Sicily [Messina] is not more than nine thousand stadia …" (G 2.1.40). Strabo recognises the inconsistency caused by the wrong courses of Eratosthenes' two meridians through *Karchedon* and the Strait of Messina and through *Alexandria* and *Rhodos*. He, however, accepts these meridians and refuses Eratosthenes' distance between *Karchedon* and *Alexandria*. In agreement with the mentioned distance of ≤ 9000 st, the sum of Strabo's distances in G 2.4.3 from the Strait of Messina to *Rhodos* is approximately 8500 st.

7.2.4 Agrippa

Pliny reports Agrippa's distance "… in a straight line from the Straits of Gades [Strait of Gibraltar] to the Gulf of Issus [*Issos*] …" (NH 6.38). Since this corresponds to the expected course of the *Rhodos* parallel, Agrippa's distances can be assumed to refer to it.

Schnabel (1935, 418–420) supposes that already Agrippa placed *Rhodos* further west than *Alexandria*. According to Schnabel (1935), the assumed meridian through promontory *Phycus* (Cape Rasat in Libya, Bostock and Riley, 1855, 4.20, n. 20), *Taenarum* (Tainaron in Laconia, S & G) and promontory *Criumetopon* (Cape Krios on Crete, S & G) was adopted by Strabo (G 17.3.20,21) from Agrippa's world map, and a comparison of the distances *Criumetopon–Rhodos* and *Phycus–Alexandria* should have led Agrippa to the conclusion that *Rhodos* is further west than *Alexandria* (strictly speaking, Strabo states that *Criumetopon* is on the meridian of *Apollonia*, which is 170 st from *Phycus*). It is, however, questionable, whether Strabo used Agrippa's world map (cf. Jones, 1917–1932, 5.2, n. 124; Dilke, 1985, 43–44), and it is unknown whether Agrippa applied the *Phycus–Taenarum–Criumetopon* meridian and which distances were available to him. Strabo would surely have mentioned such an important change of Eratosthenens' prime meridian through *Alexandria* and *Rhodos*, which is, however, not the case.

Pliny gives further longitudinal distances from Agrippa concerning the African northern coast; the two westernmost ones reach from the western end of *Mauritania Tingitana* to the Small *Syrtis* (Gulf of Gabès) and amount to 1038 and 580 Rmi (NH 5.1, 5.3; Klotz, 1931, frr. 36, 35). The sum is 12 944 st and corresponds approximately to Ptolemy's $r = 13\,133$ st from Cape *Kotes* to *Takape* (Gabès, S & G) at the Small *Syrtis*. Hence, Ptolemy's distance may originate with Agrippa.

Agrippa's $r_\mathrm{M}^\mathrm{A} = 10\,800$ st between the Strait of Messina and *Alexandria* is 2700 st smaller than Eratosthenes' value of 13 500 st and 1800 st larger than the common 9000 st to *Rhodos* reported by Strabo (see Sect. 7.2.3). Assuming that both distances were considered by Agrippa, the following cases can be distinguished. First, Agrippa used Eratosthenes' *Alexandria–Rhodos* meridian but located the Strait of Messina further east than *Karchedon*. This is less likely because then his r_M^A should be ≈ 9000 st. Second, Agrippa adopted Eratosthenes' *Karchedon–Messene* meridian and positioned *Rhodos* further west than *Alexandria* so that $9000\,\mathrm{st} < r_\mathrm{M}^\mathrm{A} \leq 13\,500$ st. Third, Agrippa did not apply the two meridians of Eratosthenes and located the Strait of Messina and *Rhodos* as described in the first and second cases, respectively, so that $9000\,\mathrm{st} < r_\mathrm{M}^\mathrm{A} < 13\,500$ st. Fourth, Agrippa adopted the two meridians of Eratosthenes, but due to the contradiction between Eratosthenes' r_M to *Alexandria* and the common 9000 st to *Rhodos*, he chose an intermediate distance. The conditions resulting from the second to fourth cases are fulfilled. According to the second and third cases, Agrippa may have located *Rhodos* further west than *Alexandria*.

8 Summary and conclusion

An answer to the disputed question of the location of Ptolemy's prime meridian at the Fortunate Islands can be found through a localisation of the places at the African western coast in GH 4.1 *Mauritania Tingitana* and GH 4.6 *Libya Interior*. The identifications of these places often differ in the literature or are missing. In the present contribution a localisation was carried out mainly based on the distances derived from the Ptolemaic coordinates.

The presumable origin of Ptolemy's coastal coordinates from the distance data of seafarings was considered including the applied measurement units and the cruising speed. The inaccuracy of the Ptolemaic distances was investigated which results from the rounding of the values of distances and coordinates and from errors concerning the ancient conversion of journey times into distances. The factor underlying the conversion between journey times and Ptolemy's distances was determined based on journey times derived from distances given by Pliny. As a result, Pliny's and Ptolemy's distances show a satisfactory match, which attests the reliability of the data.

Gosselin (1798–1813) points out that places at the African western coast are repeatedly given by Ptolemy, which is usually not recognised or not taken into account. In the present contribution, the repetition of a part of *Mauritania Tingitana* in *Libya Interior* is considered. Possibly duplicated places were selected based on their names, their relative positions and their localisations. The hypothesis of the identity of nine pairs of places was tested by means of a statistical test based on an adjustment of a transformation between the Ptolemaic coordinates. As a result, the hypothesis can be accepted. Further identical or similar places not attestable by the statistical test occur (*Erythia*/*Hera*, *Atlas*/*Mandron* Mountains), and possibly there exist more duplicates of places.

The localisation of Ptolemy's places affirmed identifications which have been supposed in different publications so far. Besides, 25 new localisations have been found (GH 4.1: nos. 2, 9, 10, 13, 23, 68, *Phokra*; GH 4.6: nos. 5–8, 10, 11, 15, 17–19, 21, 23, 24, 26–29, 96). A few important localisations shall be mentioned.

Port *Rusibis* (*Rutubis*) is usually taken for Mazagan. Ptolemy's position north of the *Asana*/Oued Oum er-Rbia and a consideration of the origination of Pliny's distances, however, lead to Casablanca or its vicinity.

Ptolemy's *Mauritania Tingitana* reaches much further south than the Roman province of the same name; the southernmost point at the coast is the Bigger *Atlas* which probably corresponds to foothills of the Anti-Atlas.

Ptolemy locates *Libya Interior* south of *Mauritania Tingitana*. Actually, the western coast of Ptolemy's *Libya Interior* begins in the north of Morocco at the *Subos*/Oued Sebou.

Cape *Arsinarion* is situated east of the Fortunate Islands and according to the Ptolemaic distances in the southwest of Morocco. Therefore, Cape *Arsinarion* is Cap Juby and the Fortunate Islands are some of the Canary Islands. The latter is substantiated by a distance by Sebosus.

It has been assumed that the Niger does not occur in ancient Greek and Latin sources (S & G, p. 449, fn. 190), but possibly it is the *Masitholos* because this river is situated between the *Hesperu Keras*/Bight of Benin and *Ochema Theon*/Mount Cameroon (and the Western High Plateau).

The single islands of the Fortunate Islands were identified based on their names, the position data of Ptolemy and Sebosus and further information from Sebosus and Juba. It shows that the ancient information is in accordance with the Canary Islands. Ptolemy's Fortunate Islands correspond to Alegranza, Graciosa, Lanzarote, Fuerteventura, Gran Canaria and Tenerife. From the ancient information follows that the smaller *Junonia* mentioned by Juba is Montaña Clara and that Sebosus' *Planasia* corresponds to Ptolemy's and Juba's *Canaria*/Gran Canaria.

Ptolemy's latitudes of the northernmost places in Morocco and of the southernmost location, the *Ochema Theon*, are nearly correct so that a determination of the latitude can be assumed in these cases. Some Ptolemaic distances between coastal places can be regarded as consistent with regard to the size of rounding errors. A few gross errors of distances were ascribed to alterations by Ptolemy. Mainly, the Ptolemaic positions show the following errors. (1) The African western coast runs in a wrong direction almost from north to south; only a few coastal stretches are oriented correctly. Reasons for this are insufficient information. (2) The places of *Libya Interior* are strongly shifted to the south. This is due to the repetition of places of *Mauritania Tingitana* in *Libya Interior* and the arrangement of *Libya Interior* south of *Mauritania Tingitana*. (3) There are coastal stretches where the Ptolemaic distances are either systematically too large or too small. Their errors are explicable, for example, by wrong assumptions about the speed in a conversion of journey times into distances due to the ignorance of ocean currents and winds. (4) Coastal distances south of the *Stacheir*/Wad As Saguia al Hamra are strongly reduced. Reasons for this may be insufficient information about the southern regions and a reduction of the available space due to the shift of *Libya Interior* to the south. (5) The distances of islands to the coast are significantly enlarged. This is a typical error of Ptolemy, which is also to be found in other regions.

In contrast to the common ancient prime meridian of *Alexandria*, Ptolemy's prime meridian is arranged at the western end of the known world, where Ptolemy located the Fortunate Islands. Hence, this prime meridian is situated at the Canary Islands or more precisely at Tenerife (*Centuria*), which is probably Ptolemy's westernmost island in reality. A comparison of arc lengths on the parallel of *Rhodos* given by ancient authors was carried out. Ptolemy's longitudinal distance of the Fortunate Islands from the westernmost continental point, the Sacred Cape, originates with Marinos. This also applies to the longitudinal distance of 7°30′ between the Fortunate Islands and the Pillars of Herakles, which was probably derived from Eratosthenes' distance between an assumed western end of Europe and the Pillars. Ptolemy's longitudinal distance from the Pillars to *Alexandria* probably arose from Eratosthenes' value of this distance and a rounding to 53°, which may have been given by Marinos. Consequently, Ptolemy's longitudinal distance of 60°30′ between the Fortunate Islands and *Alexandria* is based on Marinos' and Eratosthenes' data. From Agrippa's distance between the Strait of Messina and *Alexandria* can be derived that he possibly located *Rhodos* west of the prime meridian of *Alexandria* for the first time.

Appendix A: Cruising speed

The speed v of a seafaring can be decomposed into

$$v = v_\mathrm{s} + v_\mathrm{c} + v_\mathrm{w}, \tag{A1}$$

where v_s is the speed corresponding to the shipping performance and v_c and v_w are components resulting from the current and the wind, respectively. For v_s, the average speed $v_\varnothing = 1000\,\mathrm{st\,day^{-1}}$ (Eq. 3) is applied. Arlett (1836) reports a measured flow speed of 0.4 to $1\,\mathrm{mi\,h^{-1}}$ for regions south of 34°30′ N and of 0.5 to $0.75\,\mathrm{mi\,h^{-1}}$ for regions south of Essaouira. Thus, for v_c the value $\pm 0.7\,\mathrm{mi\,h^{-1}} \approx \pm 1.1\,\mathrm{km\,h^{-1}} \doteq \pm 155\,\mathrm{st\,day^{-1}}$ can be used depending on the direction. According to Aristeides, the speed of seafarings with favourable winds was $1200\,\mathrm{st\,day^{-1}}$, which is similar to Herodot's general value of $1300\,\mathrm{st\,day^{-1}}$ (cf. Kroll, 1921). Casson (1971, 281–296) determines speeds from ancient journey times; the average speed of journeys with favourable winds is about 5 kn, which is $1200\,\mathrm{Ist\,day^{-1}}$ and $1411\,\mathrm{Est\,day^{-1}}$. Assuming $1200\,\mathrm{st\,day^{-1}}$, the difference of $200\,\mathrm{st\,day^{-1}}$ to $v_\varnothing$ can be used for v_w of a southward voyage with favourable winds. The result is $v = 1355\,\mathrm{st\,day^{-1}}$; for calculations, the rounded down value $v_\mathrm{ns} = 1300\,\mathrm{st\,day^{-1}}$ is used here.

Ptolemy reports an assumed speed of 400 or $500\,\mathrm{st\,day^{-1}}$ in the case of changing winds (GH 1.17). From the ancient journeys with unfavourable winds investigated by Casson (1971) follows a speed of about 2 kn, i.e. $480\,\mathrm{Ist\,day^{-1}}$ and $564\,\mathrm{Est\,day^{-1}}$. Supposing $500\,\mathrm{st\,day^{-1}}$, the difference of $-500\,\mathrm{st\,day^{-1}}$ to $v_\varnothing$ can be applied to v_w of a northward voyage with adverse winds. This leads to $v = 345\,\mathrm{st\,day^{-1}}$. For the case that the conditions were not the worst, the speed $v_\mathrm{sn} = 400\,\mathrm{st\,day^{-1}}$ is set for a northward voyage.

Appendix B: Propagation of rounding errors

The spherical distance between the two Ptolemaic positions (Λ_i, Φ_i) and (Λ_j, Φ_j) is

$$\hat{s}_{i,j} = \arccos\left(\sin(\Phi_i)\sin\left(\Phi_j\right) + \cos(\Phi_i)\cos\left(\Phi_j\right)\cos\left(\Lambda_j - \Lambda_i\right)\right) \tag{B1}$$

(e.g. Gellert et al., 1967, p. 331). If $\epsilon_{\Lambda i}$, $\epsilon_{\Phi i}$, $\epsilon_{\Lambda j}$ and $\epsilon_{\Phi j}$ are the absolute values of maximum rounding errors of two positions, then the resulting error $\Delta_{\hat{s}}$ of their distance $\hat{s}$ Eq. (B1) can be estimated by means of

$$|\Delta_{\hat{s}}| \le \left|\frac{\partial \hat{s}}{\partial \Lambda_i}\right|\epsilon_{\Lambda i} + \left|\frac{\partial \hat{s}}{\partial \Lambda_j}\right|\epsilon_{\Lambda j} + \left|\frac{\partial \hat{s}}{\partial \Phi_i}\right|\epsilon_{\Phi i} + \left|\frac{\partial \hat{s}}{\partial \Phi_j}\right|\epsilon_{\Phi j} \tag{B2}$$

(cf e.g. Engeln-Müllges et al., 2005, p. 19). In GH 4.1 several coordinates were possibly rounded with a precision of 10′. If maximum simultaneous errors $\epsilon_\Lambda = \epsilon_\Phi = 3'$ are assumed and the fictitious distance from $(\Lambda_i, \Phi_i = 31°)$ to $(\Lambda_j = \Lambda_i + 30', \Phi_j = 31°30')$ with an average length is used, then $|\Delta_{\hat{s}}|$ is ≤ 12 or 10 km (Ist/Est, Eq. 10). The distances in GH 4.6 are larger and a usual resolution may be 30′. If $\epsilon_\Lambda = \epsilon_\Phi = 8'$ and the exemplary distance from $(\Lambda_i, \Phi_i = 15°)$ to $(\Lambda_j = \Lambda_i + 30', \Phi_j = 16°)$ are used, then $|\Delta_{\hat{s}}| \le 33/28$ km (Ist/Est) results.

Appendix C: Model test

The Ptolemaic coordinates are regarded as plain coordinates in the following, and the systematic differences of coordinates between GH 4.1 and GH 4.6 described in Sect. 4.4 are modelled by means of a two-dimensional transformation of coordinates (see e.g. Baumann, 1993, p. 166). This is suitable, since the decrease in the length of a degree of longitude with increasing latitude is negligible within the region of investigation and the small difference of this variation between the regions of GH 4.1 and 4.6 is accommodated by a scale factor. The transformation leads to the two condition equations

$$\Lambda_i'' + v_{\Lambda i}'' = \sin\epsilon\, a_\Phi \left(\Phi_i' + v_{\Phi i}'\right) + \cos\epsilon\, a_\Lambda \left(\Lambda_i' + v_{\Lambda i}'\right) + b_\Lambda, \tag{C1}$$

$$\Phi_i'' + v_{\Phi i}'' = \cos\epsilon\, a_\Phi \left(\Phi_i' + v_{\Phi i}'\right) - \sin\epsilon\, a_\Lambda \left(\Lambda_i' + v_{\Lambda i}'\right) + b_\Phi \tag{C2}$$

for each of the p considered pairs of places, where index $i = 1 \ldots p$. Λ_i' and Φ_i' are the start coordinates of the transformation; Λ_i'' and Φ_i'' are target coordinates. $v_{\Lambda i}'$, $v_{\Phi i}'$, $v_{\Lambda i}''$ and $v_{\Phi i}''$ are presumably random, residual differences. b_Λ and b_Φ are shift parameters of the longitudes and latitudes, a_Λ and a_Φ are their scale parameters, and ϵ is the rotation angle. The start and target coordinates are random variables. Their random components are assumed to be uncorrelated and normally distributed with standard deviations σ_Λ' and σ_Φ' (start) as well as σ_Λ'' and σ_Φ'' (target). The $u = 5$ parameters can be estimated by means of a weighted least squares adjustment. The arranged model corresponds to the Gauß–Helmert model; the solution of the estimation is determined by means of the usual procedures (see e.g. Böck, 1961).

It is assumed that the coordinates of identical places can be transformed by Eq. (C1) and that the size of the residuals is explicable by the uncertainty of the coordinates. This hypothesis is tested by means of the overall model test (e.g. Teunissen, 2006, 132–133), which tests the correctness of the adjustment model. It tests whether the estimate s_0^2 of the variance of unit weight σ_0^2 resulting from the adjustment is significantly larger than σ_0^2 (one-sided test). The test statistic is $T = f\, s_0^2/\sigma_0^2$, where $f = 2p - u$ is the number of degrees of freedom. T follows a χ_f^2 distribution if the hypothesis is correct.

For Λ_i' and Φ_i', GH 4.6 is chosen and hence GH 4.1 for Λ_i'' and Φ_i''. Investigations of other regions showed that the scale-corrected standard deviations of the Ptolemaic coordinates mostly amount to 10–20 km (cf. Kleineberg et al., 2012; Marx and Kleineberg, 2012). From the positions of GH 4.1 and GH 4.6 no high accuracy is to be expected; see

Sects. 2.2, 3.2.1. Thus, standard deviations of about 17.5 km are assumed here, which correspond to $11'$ of Ptolemy's degree (reversion of Eq. (10) based on Eq. (1) and Ist). For Λ, $11'/\cos\Phi$ is used.

In the test of the seven pairs of places given in Sect. 4.4, the *X* variants of nos. M62 (*X*: Table 2, Ω: $\Phi = 32°50'$) and L4 (*X*: $\Lambda = 9°$, $\Phi = 23°$, Ω: Table 2) yield a better fit of the coordinates so that they are used for the parameter estimation (furthermore the *X* variant of no. M6; see Sect. 4.1, no. 6). The adjustment yields $a_\Lambda = 0.30 \pm 0.03$ and $a_\Phi = 0.48 \pm 0.06$. Hence, the point cluster of GH 4.6 is strongly stretched in comparison to that of GH 4.1. This explains the longer distances and rougher resolution of the coordinate values in GH 4.6 (cf. Appendix B). It is to be expected that the random components of the coordinates are also enlarged so that σ'_Λ and σ'_Φ need to be increased. They are scaled by a factor of 2 resulting from a_Φ (it is determined geometrically more reliably than a_Λ). A new adjustment leads to $a_\Lambda = 0.30 \pm 0.02$, $a_\Phi = 0.49 \pm 0.06$, $b_\Lambda = 4°36' \pm 54'$, $b_\Phi = 22°22' \pm 1°20'$, and $\epsilon = -4°19' \pm 3°59'$. Using a significance level of $\alpha = 5\,\%$, the quantile $\chi^2_{1-\alpha,f}$ is 16.9. Since $T = 14.1 < \chi^2_{1-\alpha,f}$, the hypothesis that the places considered are identical can be accepted.

The addition of the two pairs of places given in Sect. 4.5 to the statistical test leads to $\chi^2_{1-\alpha,f} = 22.4$ and $T = 19.9$ so that identity can be assumed for these places.

Appendix D: Position of *Byzantion*

Carmody (1976) shows that Ptolemy's positions of *Alexandria*, *Lindos* (on *Rhodos*) and *Byzantion* are in a straight line if these locations are mapped by an equidistant cylindrical projection. This relationship raises the question of how the localisation of these places was carried out.

Ptolemy's latitude $\Phi = 31°$ of *Alexandria* is a rounding of the value $30°58'$ in MS 5.12, 13, which is surely the result of a gnomon measurement ($\phi = 31°12'$). The coordinates of *Lindos* (GH 5.2.34) are possibly the coordinates of the town of *Rhodos*, which is missing in the manuscripts (S & G, p. 499, fn. 50). Its $\Phi = 36°$ probably originates with Hipparchus, who has specified this value in his *Commentary on the Phenomena of Aratus and Eudoxus* 1.11.8 (see Manitius, 1894; Marx, 2015). $\Phi_B = 43°05'$ of *Byzantion* (GH 3.11.5) is certainly also based on Hipparchus, who gave a (erroneous) gnomon-shadow ratio of $120/41\,\frac{4}{5}$ (G 2.5.41) corresponding to $43°03'$. The investigation of Sect. 7 showed that Ptolemy's longitudes $\Lambda = 60°30'$ of *Alexandria* and $\Lambda = 58°40'$ of *Lindos* (*Rhodos*) were chosen according to Eratosthenes or Marinos, respectively. The remaining quantity with unknown origin is the longitude $\Lambda_B = 56°$ of *Byzantion*.

In order to simplify the determination of coordinates, Ptolemy treated geographic coordinates as if they were plain coordinates (cf. Berggren and Jones, 2000, p. 16). If Ptolemy arranged the three places in question in a straight line, then this line was determined by the positions of *Alexandria* and *Rhodos* and Λ_B was defined by the straight line and Φ_B. Possibly, Ptolemy calculated Λ_B by plane geometry and neglected the shortening of the distances of the meridians with increasing latitude by assuming a constant reduction factor for this shortening. (This factor need not be considered in the following, since its size does not affect the result.) The equation of the straight line can be written as $\Lambda = e\,\Phi + f$, where e and f are its parameters. The coordinates of *Alexandria* and *Rhodos* yield $e = -0.3\overline{6}$ and $f = 71°52'$, and Φ_B yields $\Lambda_B = 56°04'$. This value is consistent with Ptolemy's $56°$, which probably resulted from a rounding.

Appendix E: Abbreviations

c., C.	Cape(s)
D	Number of day distances
DN	Number of day-and-night seafarings
Est	Egyptian stade
G	Strabo's *Geography*
GH	*Geographike Hyphegesis*
Ist	Italian stade
L	*Libya Interior*
M	*Mauritania Tingitana*
MS	*Mathematike Syntaxis*
Mtn(s).	Mountain(s)
NH	Pliny's *Natural History*
Peripl.	Hanno's *Periplus*
r.	River
Rmi	Roman miles
S & G	Stückelberger and Graßhoff (2006)
st	stade
X	*Codex Vaticanus Graecus* 191
Ω	Ω recension

Acknowledgements. The author thanks the reviewers for valuable comments and suggestions.

Edited by: G. P. Gregori
Reviewed by: P. T. Keyser and two anonymous referees

References

Arlett, W.: Survey of some of the Canary Islands, and of part of the Western Coast of Africa, in 1835, J. Roy. Geogr. Soc. Lond., 6, 285–310, 1836.

Aujac, G., Harley, J., and Woodward, D.: 9. The Growth of an Empirical Cartography in Hellenistic Greece, in: The History of Cartography Volume One. Cartography in Prehistoric, Ancient, and Medieval Europe and the Mediterranean, The University of Chicago Press, Chicago, 148–160, 1987.

Aujac, G., Lasserre, F., Baladié, R., and Laudenbach, B. (Eds.): Strabon Géographie, Les Belles Lettres, Paris, 1969–2015.

Baumann, E.: Vermessungskunde Lehr- und Übungsbuch für Ingenieure, Band 2 Punktbestimmung nach Höhe und Lage, 4th Edn., Dümmler, Bonn, 1993.

Berger, H.: Geschichte der wissenschaftlichen Erdkunde der Griechen, 2nd Edn., Verlag Von Veit & Comp., Leipzig, 1903.

Berggren, J. and Jones, A.: Ptolemy's Geography: An Annotated Translation of the Theoretical Chapters, Princeton University Press, Princeton, 2000.

Blair, J.: The History of the Rise and Progress of Geography, London, 1784.

Böck, R.: Allgemeinste Formulierung der Ausgleichungsrechnung nach der Methode der kleinsten Quadratsummen, Z. Vermessungsw., 86, 43–45, 98–106, 1961.

Bostock, J. and Riley, H. (Eds.): The Natural History. Pliny the Elder, Taylor and Francis, London, 1855.

Brodersen, K. (Ed.): C. Plinius Secundus d. Ä. Naturkunde Lateinisch–deutsch Buch VI Geographie: Asien, Artemis & Winkler, Zürich, 1996.

Buch, L.: Allgemeine Uebersicht der Flora auf den Canarischen Inseln, in: Abhandlungen der Königlichen Akademie der Wissenschaften in Berlin. Aus den Jahren 1816–1817, Realschul-Buchhandlung, Physikalische Klasse, Berlin, 337–382, 1819.

Bunbury, E.: A History of Ancient Geography Among the Greeks and Romans From the Earliest Ages Till the Fall of the Roman Empire, in: vol. 1, John Murray, London, 1879.

Burton, R.: Account of the Ascent of the Camaroons Mountain, in Western Africa, in: Proceedings of the Royal Geographical Society, John Murray, London, 238–48, 1862.

Carmody, F.: Ptolemy's Triangulation of the Eastern Mediterranean, Isis, 67, 601–609, 1976.

Casson, L.: Ships and Seamanship in the Ancient World, Princeton University Press, Princeton, 1971.

Chisholm, H. (Ed.): Encyclopaedia Britannica, 11th Edn., Cambridge University Press, 1911.

Culican, W.: Phoenicia and Phoenician colonization, in: The Cambridge ancient history Vol. 3 Pt. 2 The Assyrian and Babylonian empires and other states of the Near East, from the eighth to the sixth centuries b.c., Cambridge University Press, 1991.

de Smet, K.: Status of the Nile crocodile in the Sahara desert, Hydrobiologia, 391, 81–86, 1998.

Dicks, D.: The Geographical Fragments of Hipparchus, The Athlone Press, London, 1960.

Dilke, O.: Greek and Roman Maps, Thames and Hudson, London, 1985.

Engeln-Müllges, G., Niederdrenk, K., and Wodicka, R.: Numerik-Algorithmen: Verfahren, Beispiele, Anwendungen, 9th Edn., Springer, Berlin, 2005.

Euzennat, M.: Zilis, in: The Princeton Encyclopedia of Classical Sites, Princeton University Press, Princeton, 1976.

Fischer, C.: Fortunatae insulae, in: Real-Encyclopädie der classischen Altertumswissenschaft, vol. VII, J. B. Metzler, Stuttgart, 1910.

Forbiger, A.: Handbuch der alten Geographie, vol. 1, Verlag von Mayer und Wigand, Leipzig, 1842.

Forbiger, A.: Handbuch der alten Geographie, vol. 2, Verlag von Mayer und Wigand, Leipzig, 1844.

Gellert, W., Küstner, H., Hellwich, M., and Kästner, H. (Eds.): Kleine Enzyklopädie Mathematik, 2nd Edn., VEB Bibliographisches Institut, Leipzig, 1967.

Google Inc.: Google Earth, version 7, available at: http://www.google.com/earth/, 2015.

Gosselin, P.: Recherches sur la géographie systématique et positive des anciens, in: vol. 1, Imprimerie de la République, Paris, 1798–1813.

Gyory, J., Mariano, A., and Ryan, E.: The Canary Current, http://oceancurrents.rsmas.miami.edu/atlantic/canary.html (last access: 3 Juny 2015), 2013.

Hennig, R.: Terrae Incognitae, in: Vol. 1, 2nd Edn., E. J. Brill, Leiden, 1944.

Hughes, R., Hughes, J., and Bernacsek, G.: A Directory of African Wetlands, IUCN, Gland, 1992.

Hultsch, F.: Griechische und Römische Metrologie, 2nd Edn., Weidmannsche Buchhandlung, Berlin, 1882.

Jones, H. (Ed.): The Geography of Strabo, Loeb Classical Library series, Harvard University Press, Cambridge, 1917–1932.

Keyser, P.: From Myth to Map: The Blessed Isles in the First Century B.C., Ancient World, 24, 149–168, 1993.

Kleineberg, A., Marx, C., and Lelgemann, D.: Europa in der Geographie des Ptolemaios. Die Entschlüsselung des "Atlas der Oikumene": Zwischen Orkney, Gibraltar und den Dinariden, Wissenschaftliche Buchgesellschaft, Darmstadt, 2012.

Klotz, A.: Die geographischen commentarii des Agrippa und ihre Überreste, Klio, 24, 386–466, 1931.

Kroll, W.: Schiffahrt, in: Real-Encyclopädie der classischen Altertumswissenschaft, in: Vol. II A, J. B. Metzler, Stuttgart, 1921.

Krüss, J.: The names of the Canary Islands and their verification, in: Biogeography and Ecology in the Canary Islands, Dr. W. Junk bv Publishers, The Hague, 1976.

Lacroix, W.: Africa in Antiquity. A Linguistic and Toponymic Analysis of Ptolemy's Map of Africa, together with a Discussion of Ophir, Punt and Hanno's Voyage, Verlag für Entwicklungspolitik, Saarbrücken, 1998.

Lendering, J.: Hanno the Navigator, http://www.livius.org/articles/person/hanno-1-the-navigator/ (last access: 4 December 2014), 1998–2014.

Lockwood, J.: Atmospheric Circulation, global, in: The Encyclopedia of World Climatology, Springer, Dordrecht, 2005.

Manitius, K. (Ed.): Hipparchi in Arati et Eudoxi Phaenomena Commentariorum Libri Tres, B. G. Teubner, Leipzig, 1894.

Mannert, K.: Geographie der Griechen und Römer. Afrika, Hahnsche Verlags-Buchhandlung, Leipzig, 1825.

Marx, C.: On the precision of Ptolemy's geographic coordinates in his Geographike Hyphegesis, His. Geo-Space Sci., 2, 29–37, 2011.

Marx, C.: Rectification of the ancient geographic coordinates in Ptolemy's Geographike Hyphegesis, Hist. Geo-Space Sci., 3, 99–112, 2012.

Marx, C.: An analysis of the latitudinal data of Eratosthenes and Hipparchus, Math. Mech. Complex Syst., 3, 309–339, 2015.

Marx, C. and Kleineberg, A.: Die Geographie des Ptolemaios. Geographike Hyphegesis Buch 3: Europa zwischen Newa, Don und Mittelmeer, epubli, Berlin, 2012.

Mauny, R.: The Failure To Make Maritime Contact With West Africa, in: Cambridge History of Africa, Vol. 2, Cambridge University Press, Cambridge, 1978.

Müller, C.: Studien zur Geschichte der Erdkunde im Altertum. I. Die Kunde des Altertums von den Canarischen Inseln. II. Statius Sebosus, Breslauer Genossenschafts-Buchdruckerei, Breslau, 1902.

Pinkerton, J.: Modern Geography, in: Vol. 2, new Edn., Cadell and Davies, London, 1817.

Pothecary, S.: Strabo, Polybios, and the Stade, Phoenix, 49, 49–67, 1995.

Roller, D.: Eratosthenes' Geography, Princeton University Press, Princeton, 2010.

Romer, F.: Pomponius Mela's Description of the World, The University of Michigan Press, Ann Arbor, 1998.

Schnabel, P.: Die Weltkarte des Agrippa als wissenschaftliches Mittelglied zwischen Hipparch und Ptolemaeus, Philologus, 90, 405–440, 1935.

Spaul, J.: Studies in the Roman province of Mauretania Tingitana. Part 1 The Geography of Tingitana, Durham University, available at: http://etheses.dur.ac.uk/10374/, 1958.

Stückelberger, A. and Graßhoff, G. (Eds.): Klaudios Ptolemaios Handbuch der Geographie, Schwabe, Basel, 2006.

Talbert, R. (Ed.): Barrington Atlas of the Greek and Roman World, Princeton University Press, Princeton, 2000.

Teunissen, P.: Testing theory an introduction, 2nd Edn., VSSD, Delft, 2006.

Thomson, J.: History of Ancient Geography, Cambridge University Press, Cambridge, 1948.

Toomer, G. (Ed.): Ptolemy's Almagest, reprint 1998, Princeton University Press, Princeton, 1984.

Winkler, G. and König, R. (Eds.): C. Plinius Secundus d. Ä. Naturkunde Lateinisch–deutsch Buch V Geographie: Afrika und Asien, Artemis & Winkler, München, 1993.